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RESULTS ON THE EXPONENTIAL INTEGRAL

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Abstract. Results on the convolution product of the exponential integral and exponential function are given. These results are found in a space of distributions. The convolution products gained in this work may be considered as a generalization of Chandrasekhar's functions which are needed in the problem of diffuse reflections and transmission of radiation by an atmosphere.

1. Introduction

The exponential integral $\mathrm{ei}(x) = E_1(x)$ is defined for $x > 0$ by

$$\mathrm{ei}(x) = \int_x^\infty u^{-1}e^{-u}du = \int_1^\infty u^{-1}e^{-ux}du\,, \tag{1.1}$$

see [2], and the integral is diverging for $x \leq 0$. This integral frequently arises in many fields of physics and engineering. For example, we would like to say that there are connections of exponential integral and some integrals in quantum chemistry, the exponential integral has application in fluid dynamics and transport theory, and it is applied to the solution of Milne's integral equations. Moreover, this integral play an important role in the theory of nuclear reactions, see [5].

The exponential integral has its generalization $E_n(x)$ for $x > 0$ given by the equation:

$$E_n(x) = \int_1^\infty u^{-n}e^{-ux}du\,, \quad n = 0, 1, 2\dots \tag{1.2}$$

The generalized exponential integral (1.2) is one of the most fundamental integrals in engineering, specifically in antenna theory, and for many years exact solutions to this integral have been sought. Some exact solutions to

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the generalized exponential integral which is completely general and independent of the usual restrictions involving the wavelength, field point distance and dipole length are given in [7]. The authors of [7] use the exact expression for the generalized exponential integral in their computation of the impedance matrix elements, and they demonstrated that, for very thin straight-wire antennas, an asymptotic expansion can be used to obtain a numerically convenient form of the generalized exponential integral.

So in this paper we evaluate convolution products of distributions. The theory of distribution was first formulated by Sobolev and then developed systematically by L. Schwartz, [1]. It is a subject that is used not only in many mathematical disciplines such as functional analysis and applied mathematics, see [8, 4, 17, 18, 19] but also in physics and engineering science, see [9, 10]. Together with Schwartz's theory, some singular functions such as the Dirac delta function δ and operations with them can be defined mathematically.

2. Notations

In this paper we will evaluate some convolution product of the exponential integral and exponential function. It was pointed out in [11] that the equation (1.1) can be rewritten in the form

$$\mathrm{ei}(x) = \int_x^\infty u^{-1}\left[e^{-u} - H(1-u)\right] du - H(1-x)\ln|x|\,, \tag{2.1}$$

where H denotes Heaviside's function. The integral in this equation is convergent for all x and we use the definition given with (2.1) to define $\mathrm{ei}(x)$ on the real line.

Further, we define the function $\mathrm{ei}_+(x)$ and $\mathrm{ei}_-(x)$ by

$$\mathrm{ei}_+(x) = H(x)\,\mathrm{ei}(x)\,, \quad \mathrm{ei}_-(x) = H(-x)\,\mathrm{ei}(x)$$

so that

$$\mathrm{ei}(x) = \mathrm{ei}_+(x) + \mathrm{ei}_-(x)\,.$$

We can extend this definition by adding a parameter $\lambda > 0$, and more generally, see [12], we define $\mathrm{ei}(\lambda x)$, $\mathrm{ei}_+(\lambda x) = H(x)\,\mathrm{ei}(\lambda x)$ and $\mathrm{ei}_-(\lambda x) = H(-x)\,\mathrm{ei}(\lambda x)$ in the obvious way by:

$$\begin{aligned}
\mathrm{ei}(\lambda x) &= \int_x^\infty u^{-1}\left[e^{-\lambda u} - H(1-\lambda u)\right] du - H(1-\lambda x)\ln|\lambda x|\,,\\
\mathrm{ei}_+(\lambda x) &= \int_x^\infty u^{-1} e^{-\lambda u} du\,,\ x > 0\,,\\
\mathrm{ei}_-(\lambda x) &= -\gamma(\lambda) + \int_x^0 u^{-1}\left(e^{-\lambda u} - 1\right) du - \ln x_-\,,\ x < 0\,,
\end{aligned} \tag{2.2}$$

where $\gamma(\lambda) = \gamma + \ln|\lambda|$ and $\gamma = -\int_0^\infty u^{-1}\left(e^{-u} - H(1-u)\right) du$, is Euler's constant. Also it is clear that the relation

$$\mathrm{ei}(\lambda x) = \mathrm{ei}_+(\lambda x) + \mathrm{ei}_-(\lambda x)\,,$$

holds, and in particular we have that:

$$\mathrm{ei}_-(x) = -\gamma + \int_x^0 u^{-1}\left(e^{-u} - 1\right) du - \ln x_-\,,\ x < 0\,, \tag{2.3}$$

$$\mathrm{ei}_-(2x) = -\gamma(2) + \int_x^0 u^{-1}\left(e^{-2u} - 1\right) du - \ln x_-\,,\ x < 0\,. \tag{2.4}$$

where $\gamma = -\int_0^\infty u^{-1}\left[e^{-u} - H(1-u)\right] du$ is Euler's constant, and $\gamma(2) = \gamma + \ln 2$. The locally summable functions e_+^x and e_-^x are defined by:

$$e_+^x = H(x)e^x\,, \quad e_-^x = H(-x)e^x\,.$$

The derivatives of these functions are given by:

$$\begin{aligned}
[\mathrm{ei}(\lambda x)]' &= -x^{-1}e^{-\lambda x}\,,\\
[\mathrm{ei}_+(\lambda x)]' &= -x_+^{-1}e^{-\lambda x} - \gamma(\lambda)\delta(x)\,,\\
[\mathrm{ei}_-(\lambda x)]' &= x_-^{-1}e^{-\lambda x} + \gamma(\lambda)\delta(x)\,,
\end{aligned} \tag{2.5}$$

and for $\lambda = 1$ we have:

$$\begin{aligned}
[\mathrm{ei}(x)]' &= -x^{-1}e^{-x}\,,\\
[\mathrm{ei}_+(x)]' &= -x_+^{-1}e^{-x} - \gamma\delta(x)\,,\\
[\mathrm{ei}_-(x)]' &= x_-^{-1}e^{-x} + \gamma\delta(x)\,,
\end{aligned} \tag{2.6}$$

3. Convolution product

In the following we let $\mathcal{D}$ be the space of infinitely differentiable functions with compact support and let $\mathcal{D}'$ be the space of distributions defined on $\mathcal{D}$.

The classical definition of the convolution product of two functions f and g is as follows:

Definition 3.1. Let f and g be functions. Then the *convolution* $f * g$ is defined by

$$(f * g)(x) = \int_{-\infty}^{\infty} f(t)g(x-t)\,dt$$

for all points x for which the integral exist.

It follows easily from the definition that if $f * g$ exists then $g * f$ exists and

$$f * g = g * f \tag{3.1}$$

and if $(f*g)'$ and $f*g'$ (or $f'*g$) exists, then

$$(f*g)' = f*g' \quad (\text{or } f'*g). \tag{3.2}$$

Definition 3.1 can be extended to define the convolution $f*g$ of two distributions f and g in $\mathcal{D}'$ with the following definition, see Gel'fand and Shilov [3].

Definition 3.2. Let f and g be distributions in $\mathcal{D}'$. Then the *convolution* $f*g$ is defined by the equation

$$\langle (f*g)(x), \varphi \rangle = \langle f(y), \langle g(x), \varphi(x+y) \rangle \rangle$$

for arbitrary φ in $\mathcal{D}$, provided f and g satisfy either of the conditions
(a) either f or g has bounded support,
(b) the supports of f and g are bounded on the same side.

It follows that if the convolution $f*g$ exists by this definition then equations (3.1) and (3.2) are satisfied.

The problem of finding convolution products of distributions has been exploited by many researchers, since in general this product does not always exists. In this paper the convolution of $\mathrm{ei}_-(\lambda x)$ and e^x_- , and the convolution of $x\,\mathrm{ei}_-(\lambda x)$ and e^x_- are evaluated. At the end the convolutions of their derivatives are given. Similar results on convolution products and neutrix convolution products can be found in [20, 13, 14, 15, 16].

4. Main results

Theorem 4.1. The convolution product $\mathrm{ei}_-(\lambda x) * e^x_-$ exists and

$$\mathrm{ei}_-(\lambda x) * e^x_- = \mathrm{ei}_-(\lambda x) - e^x\,\mathrm{ei}_-((\lambda+1)x) + \ln\left|\frac{\lambda}{\lambda+1}\right| e^x_- \,, \tag{4.1}$$

for $\lambda > 0$.

In particular, we have:

$$\mathrm{ei}_-(x) * e^x_- = \mathrm{ei}_-(x) - e^x\,\mathrm{ei}_-(2x) - \ln 2 e^x_- \,, \tag{4.2}$$

for $\lambda = 1$.

Proof. The convolution $\mathrm{ei}_-(\lambda x) * e^x_- = 0$ if $x > 0$, so we have:

$$\begin{aligned} \mathrm{ei}_-(\lambda x) * e^x_- &= \int_x^0 \mathrm{ei}_-(\lambda t) e^{x-t}\,dt \\ &= e^x \int_x^0 \left[-\gamma(\lambda) + \int_t^0 u^{-1}\left(e^{-\lambda u} - 1\right) du - \ln t_- \right] e^{-t}\,dt \\ &= I_1 + I_2 + I_3 \,. \end{aligned} \tag{4.3}$$

Now we have:

$$I_1 = -\gamma(\lambda) e^x \int_x^0 e^{-t} = \gamma(\lambda)\left(e^x - 1\right) . \tag{4.4}$$

$$\begin{aligned} I_3 &= -e^x \int_x^0 e^{-t} \ln t_- \, dt \\ &= -e^x \left\{ e^{-x} \ln x_- + \int_x^0 t^{-1}\left(e^{-t} - 1\right) \, dt - \ln x_- \right\} \\ &= -e^x \left(e^{-x} \ln x_- + \gamma + \mathrm{ei}_-(x)\right) = -\gamma e^x - e^x \, \mathrm{ei}_-(x) - \ln x_- . \end{aligned} \tag{4.5}$$

$$\begin{aligned} I_2 &= e^x \int_x^0 e^{-t} \int_t^0 u^{-1}(e^{-\lambda u} - 1) \, du \, dt = e^x \int_x^0 u^{-1}(e^{-\lambda u} - 1) \int_x^u e^{-t} \, dt \, du \\ &= e^x \int_x^0 u^{-1}(e^{-\lambda u} - 1)(e^{-x} - e^{-u}) \, du \\ &= \int_x^0 u^{-1}(e^{-\lambda u} - 1) \, du + e^x \int_x^0 u^{-1}(e^{-u} - 1) du - e^x \int_x^0 u^{-1}(e^{-(\lambda+1)u} - 1) du. \end{aligned}$$

Using equations (2.3) and (2.2) for I_2 integral we have that:

$$I_2 = \gamma(\lambda) + \mathrm{ei}_-(\lambda x) + e^x \left\{ \mathrm{ei}_-(x) - \mathrm{ei}_-((\lambda+1)x) + \gamma - \gamma(\lambda+1) \right\} + \ln x_- . \tag{4.6}$$

Finally equation (4.1) follows from equations (4.4), (4.6) and (4.5). Equation (4.2) follows from equation (4.1) and (2.4) for $\lambda = 1$, proving the theorem. □

Theorem 4.2. The convolution product $x \, \mathrm{ei}_-(\lambda x) * e_-^x$ exists and

$$\begin{aligned} x \, \mathrm{ei}_-(\lambda x) * e_-^x &= (x+1) \, \mathrm{ei}_-(\lambda x) - e^x \, \mathrm{ei}_-((\lambda+1)x) \\ &\quad + \left(\ln\left| \frac{\lambda}{\lambda+1} \right| + \frac{1}{\lambda+1} \right) e_-^x - 2(x+1) \ln x_- - \frac{1}{\lambda+1} e^{-\lambda x} - 1 , \end{aligned} \tag{4.7}$$

for $\lambda > 0$.

In particular, we have:

$$\begin{aligned} x \, \mathrm{ei}_-(x) * e_-^x &= (x+1) \, \mathrm{ei}_-(x) - e^x \, \mathrm{ei}_-(2x) \\ &\quad + \left(\frac{1}{2} - \ln 2\right) e_-^x - 2(x+1) \ln x_- - \frac{1}{2} e^{-x} - 1 , \end{aligned} \tag{4.8}$$

for $\lambda = 1$.

Proof. The convolution $x\,\mathrm{ei}_-(\lambda x) * e^x_- = 0$ if $x > 0$, so we suppose that $x < 0$. Then:

$$\begin{aligned} x\,\mathrm{ei}_-(\lambda x) * e^x_- &= \int_x^0 t\,\mathrm{ei}_-(\lambda t) e^{x-t}\,dt \\ &= e^x \int_x^0 \left[-\gamma(\lambda) + \int_t^0 u^{-1}\left(e^{-\lambda u} - 1\right)\,du - \ln t_-\right] te^{-t}\,dt \\ &= J_1 + J_2 + J_3\,. \end{aligned} \tag{4.9}$$

Now we have:

$$J_1 = -\gamma(\lambda) e^x \int_x^0 te^{-t} = \gamma(\lambda)\left(e^x - x - 1\right)\,. \tag{4.10}$$

$$\begin{aligned} J_3 &= -e^x \int_x^0 te^{-t} \ln t_-\,dt \\ &= -e^x \left\{ xe^{-x}\ln x_- + \int_x^0 e^{-t}\ln t\,dt + \int_x^0 e^{-t}\,dt \right\} \\ &= -e^x \left(xe^{-x}\ln x_- + e^{-x}\ln x_- + \gamma + \mathrm{ei}_-(x) - 1 + e^{-x} \right) \\ &= e^x\left(1 - \gamma - \mathrm{ei}_-(x)\right) - 1 - (x+1)\ln x_-\,. \end{aligned} \tag{4.11}$$

$$\begin{aligned} J_2 &= e^x \int_x^0 te^{-t} \int_t^0 u^{-1}(e^{-\lambda u} - 1)\,du\,dt = e^x \int_x^0 u^{-1}(e^{-\lambda u} - 1) \int_x^u te^{-t}\,dt\,du \\ &= e^x \int_x^0 u^{-1}(e^{-\lambda u} - 1)\left[(x+1)e^{-x} - (u+1)e^{-u}\right]\,du \\ &= (x+1)\int_x^0 u^{-1}(e^{-\lambda u} - 1)\,du - e^x \int_x^0 \left(1 + u^{-1}\right)\left(e^{-(\lambda+1)u} - e^{-u}\right)\,du\,. \end{aligned}$$

Using equations (2.3) and (2.2) for J_2 integral we have that:

$$\begin{aligned} J_2 = (1+x)\,\mathrm{ei}_-(\lambda x) + e^x\left(\mathrm{ei}_-(x) - \mathrm{ei}_-((\lambda+1)x)\right) + e^x\left(\gamma - \gamma(\lambda+1) - 1\right) \\ + (x+1)\left(\gamma(\lambda) - \ln x_-\right) - \frac{e^x}{\lambda+1}\left(e^{-(\lambda+1)x} - 1\right) + 1\,. \end{aligned} \tag{4.12}$$

Finally equation (4.7) follows from equations (4.10), (4.12) and (4.11). Equation (4.8) follows from equation (4.7) and (2.4) for $\lambda = 1$, proving the theorem. □

Corollary 4.1. The convolution product $\mathrm{ei}_-(\lambda x) * xe_-^x$ exists and

$$\begin{aligned}\mathrm{ei}_-(\lambda x) * xe_-^x &= (1-x)\, e^x \,\mathrm{ei}_-((\lambda+1)x) - \mathrm{ei}_-(\lambda x) \\ &+ \left((x-1)\ln\left|\frac{\lambda}{\lambda+1}\right| + \frac{1}{\lambda+1}\right) e_-^x + 2(x+1)\ln x_- + \frac{1}{\lambda+1} e^{-\lambda x} + 1\,,\end{aligned} \tag{4.13}$$

for $\lambda > 0$.

In particular, we have:

$$\begin{aligned}\mathrm{ei}_-(x) * xe_-^x &= (1-x)\, e^x \,\mathrm{ei}_-(2x) - \mathrm{ei}_-(x) \\ &+ \left(\frac{1}{2} + \ln 2(1-x)\right) e_-^x + 2(x+1)\ln x_- + \frac{1}{2} e^{-x} + 1\,,\end{aligned} \tag{4.14}$$

for $\lambda = 1$.

Proof. The convolution $\mathrm{ei}_-(\lambda x) * xe_-^x = 0$ if $x > 0$, so we have:

$$\begin{aligned}\mathrm{ei}_-(\lambda x) * xe_-^x &= \int_x^0 \mathrm{ei}_-(\lambda t)(x-t)e^{x-t}\, dt \\ &= x\int_x^0 \mathrm{ei}_-(\lambda t)e^{x-t}\, dt - \int_x^0 \mathrm{ei}_-(\lambda t)te^{x-t}\, dt\,.\end{aligned}$$

Now using equation (4.3) for the first integral above and equation (4.9) for the second integral we prove the (4.13). From (4.13) for $\lambda = 1$ equation (4.14) follows immediately. □

Corollary 4.2. The convolution product $x_-^{-1}e^{-\lambda x} * e_-^x$ exists and

$$x_-^{-1}e^{-\lambda x} * e_-^x = -e^x\,\mathrm{ei}_-((\lambda+1)x) + \left[\ln\left|\frac{\lambda}{\lambda+1}\right| - \gamma(\lambda)\right] e_-^x\,, \tag{4.15}$$

for $\lambda > 0$.

In particular, we have:

$$x_-^{-1}e^{-x} * e_-^x = -e^x\,\mathrm{ei}_-(2x) + \gamma(2)e_-^x\,. \tag{4.16}$$

Proof. The convolution product $x_-^{-1}e^{-\lambda x} * e_-^x$ exists by Definition 3.2, since $x_-^{-1}e^{-\lambda x}$ and e_-^x are bounded from the right. Using equations (3.2) and (2.5), we have:

$$\begin{aligned}\left[\mathrm{ei}_-(\lambda x) * e_-^x\right]' &= \left[x_-^{-1}e^{-\lambda x} + \gamma(\lambda)\delta(x)\right] * e_-^x \\ &= x_-^{-1}e^{-\lambda x} * e_-^x - \gamma(\lambda)e_-^x \\ &= \mathrm{ei}_-(\lambda x) * \left[e_-^x - \delta(x)\right].\end{aligned}$$

Using the equation (4.1), the equation (4.15) follows immediately. From (4.15) for $\lambda = 1$ we prove the equation (4.16). □

In the corollary 4.3, the distribution x_-^{-2} i defined by $x_-^{-2} = \left(-x_-^{-1}\right)'$ and not as in Gel'fand and Shilov, see [3].

Corollary 4.3. The convolution product $x_-^{-2}e^{-\lambda x} * e_-^x$ exists and

$$x_-^{-2}e^{-\lambda x} * e_-^x = x_-^{-1}e^{-\lambda x} + (1+\lambda)e^x \operatorname{ei}_-((\lambda+1)x) - (1+\lambda)\left[\ln\left|\frac{\lambda}{\lambda+1}\right| - \gamma(\lambda)\right]e_-^x\,, \quad (4.17)$$

for $\lambda > 0$.

In particular, we have:

$$x_-^{-2}e^{-x} * e_-^x = x_-^{-1}e^{-x} + 2e^x \operatorname{ei}_-(2x) - 2\gamma(2)e_-^x\,. \quad (4.18)$$

Proof. The convolution product $x_-^{-2}e^{-\lambda x} * e_-^x$ exists by Definition 3.2, since $x_-^{-2}e^{-\lambda x}$ and e_-^x are bounded from the right. Using equation (3.2), we have:

$$\begin{aligned}\left[x_-^{-1}e^{-\lambda x} * e_-^x\right]' &= -\left[x_-^{-2}e^{-\lambda x} + \lambda x_-^{-1}e^{-\lambda x}\right] * e_-^x \\ &= -x_-^{-2}e^{-\lambda x} * e_-^x - \lambda x_-^{-1}e^{-\lambda x} * e_-^x \\ &= x_-^{-1}e^{-\lambda x} * \left[e_-^x - \delta(x)\right] \\ &= x_-^{-1}e^{-\lambda x} * e_-^x - x_-^{-1}e^{-\lambda x}\,.\end{aligned}$$

Using the equation (4.15), the equation (4.17) follows immediately. From (4.17) for $\lambda = 1$ we prove the equation (4.18). □

5. Conclusions

In this work we present a collection of convolution products of the exponential integral and exponential function. These results are original, to our knowledge, and are obtained using the theory of distributions. The convolution products gained in this work may be considered as a generalization of Chandrasekhar's functions, see [6], which are the moments of the functions needed in the problem of diffuse reflections and transmission of radiation by an atmosphere.

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