

**THE CONTRIBUTIONS AND LEGACY OF ONE
OF THE GREATEST LOMONOSOV ALGEBRAISTS,
Professor Alexander Vasilevich Mikhalev**



(Bryansk, November 8, 1940 – Moscow, October 27, 2022)

Professor Alexander Vasilevich Mikhalev a prominent Russian and Soviet mathematician, passed away on October 27, 2022. He left a great emptiness in the world of algebra and computer science as well as for everyone who knew this magnificent researcher and man.

He was one of the most significant representatives of the *Moscow School of Algebra and Computer Science*, and was one of the last giants – a real scientific rock of the *Department of Higher Algebra of Lomonosov Moscow State University*.

Alexander Vasilevich Mikhalev was born just before the *Second World War* on November 8, 1940 in Bryansk, Russia. In his childhood, he survived the difficult war years, poverty and evacuation. As a very small child, he helped the wounded in the hospital and encouraged them by reading them the news. That clever little boy never forgot the brave fighters and their sufferings. Even as a child, he understood what fascism was and stood on the side of his Motherland, where he remained throughout his life.

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He graduated from high school with a *gold medal*. When he was only 15 years old he enrolled in the *Faculty of Mechanics and Mathematics of Lomonosov Moscow State University* (MSU) to which he remained attached throughout his life. As one of the best students of his generation, he graduated with honours – “*red diploma*” (and excellent grades in all subjects) in 1961 and continued his education at the *Department of Higher Algebra*. His first steps in scientific research were made under the guidance of Professor *Lev Antol'evich Skorniakov*. There he completed his Candidate's thesis: “*Isomorphisms of semigroups of ring endomorphisms*” (1967) which then was equivalent to a doctoral thesis in the Western world. Before that, he had a one-year study stay at Columbia University (New York, 1965-66), working under the direction of two mathematicians: *Samuel Eilenberg and Saunders Mac Lane*. At MSU he completed his Doctoral thesis: “*Endomorphisms of moduls and multiplicative rings construction*” in 1990. At that time in the USSR, a doctorate was earned for a lifetime achievement.

Alexander Vasilevich met his wife Ludmila Ivanovna during his student days. They had a very long and wonderful marriage. Their son Alexander Alexandrovich is also a very successful mathematician.

Alexander Vasilevich worked at Moscow State University – Lomonosov for more than 50 years, first as an assistant (1966), then an assistant professor (1970), and professor of the Department of Higher Algebra from 1992 until the end of his life. In fact he worked at Lomonosov during his entire working life.

He published about 500 works, including 300 scientific original papers and 55 review articles, 24 monographs and 10 textbooks. He also translated several fundamental scientific monographs into Russian and delivered many fundamental courses and lectures, while also introducing and founding completely new courses.

Besides, he was the research supervisor to 112 seekers of a Candidate degree and about 20 candidates for a Doctor Degree in mathematical sciences. A number of well-known open problems were solved by him and his students and new scientific directions in mathematics and informatics were built. Now his students are actively working in the higher education system and scientific research both in their homeland and around the world.

In addition to being a professor at the *High Algebra Department* he was the *Head of the Laboratory for Applied Methods* of the Faculty of Mechanics and Mathematics of MSU (1979- 2014), as well as *Head of the Department of Theoretical Informatics* from 2013 until the end of his life. Also he was the Vice-Rector of the *Moscow State University* from 1999 to 2009.

Alexander Vasilevich Mikhalev's field of scientific work was very broad, but, his basic scientific interests were related to algebra and informatics. In algebra he achieved significant contributions, especially in: ring and module theory, homological algebra, algebraical K -theory, linear groups, subgroups, topological groups and orderings, differential algebra, commutative algebra, while in computer science he worked in: computer algebra, theoretical informatics, coding theory and cryptography.

He also worked in other areas of mathematics, such as: algebraic theory of measure, theory of modules and foundations of mathematics, mathematical modeling, etc.

Prof. Mikhalev was a world class mathematician who during his long and distinguished career made immense contributions to research and teaching mathematics. Although it is

very difficult to present A. V. Mikhalev as a mathematician in only a few brief words, we will attempt to describe his contributions.

Among his numerous results, we will mention only some of the most significant ones. He solved alone, or together with his students, a large number of problems that had been open for a long time: *the Baer-Kaplansky problem*; *the Schreier-Van der Waerden problem* (with I.Z. Golubchik); *the Herstein's problem* (with K.I. Beidar, W. Martindale); *the Riesz-Radon problem* on (with V.K. Zakharov); *the Maltsev's problems* (with K.I. Beidar and E.I. Bunina).

Alexander Vasilevich was not only a solver of problems, but also a builder of many algebraic theories. These include: *multiplicative classification of rings*; *the theory of orthogonally complete algebraic systems with applications in the theory of rings and modules* (with K.I. Beidar); *theory of dimensional differential and difference polynomials of Hilbert-Einstein-Klochin* (with E.V. Pankratiev, A.V. Levin); *homological classification of monoids* (with L.A. Skorniyakov, W. Knauer); *extension of topologies to polynomial rings, semigroup and group rings* (with V.I. Arnautov); *codes and recurrent sequences over rings and modules* (with A.A. Nechaev); *theory of mathematical systems and levels of the foundation of mathematics* (with V.K. Zakharov); *construction of mathematical models of air traffic control*.

Besides, Prof. Mikhalev was the editor-in-chief of several series of monographs and textbooks "*Fundamentals in Informatics and Mathematics*" and the journal "*Fundamental and Applied Mathematics*" (in Russian) published by Springer as "*Journal of Mathematical Sciences*" (in English), as well as a member of several editorial boards of leading Russian and international mathematical journals, including: "*Proceedings of the I. G. Petrovsky Seminar*", "*Mathematical questions in cryptography*", "*Chebyshev's Proceedings*", "*Abelian Groups and Modules*", "*Southeast Asian Bulletin of Mathematics*", "*Algebra and Discrete Mathematics*", "*Asian-European Journal of Mathematics*", "*Izvestia of the Academy of Sciences of the Republic of Moldova*", "*Discussiones Mathematicae – General Algebra and Applications*", "*Groups – Complexity – Cryptology*". For many years he worked for the Russian Reviews Journal "*MATEMATIKA*" VINITI of the Russian Academy of Sciences (RAN). Alexander Vasilevich served also as an editor of our journal, the "*Sarajevo Journal of Mathematics*".

Alexander Vasilevich Mikhalev received numerous recognitions and awards for his long-term services in science and education. Thus, he was a *Laureate of the USSR Council of Ministers Prize for Applied Research* (1982), an *Honoured Scientist of the Russian Federation* (2003), a *Laureate of the M.V. Lomonosov Prize* (2004), an *Acting member of the Professional Higher Education of the Russian Federation* (2005), an *Honoured Professor of the MSU* (2005), a *Laureate of the Prize of the President of the Russian Federation in Domain of Education* (2005), etc.

As a result of such rich scientific, professional and other activities Alexander Vasilevich, was elected as a member of the *International Academy of Sciences* of the Higher School in 1996 and as a member of the *Russian Academy of Natural Sciences* in 2002.

Perhaps the two main and most significant subjects of A. V. Mikhalev's work can be summarised as:

1. THE THEORY OF MODULES (THE MAIN SUBJECT OF THE WORK OF A. MIKHALEV)

Building blocks in mathematics are the group representations, and in number theory these are *Galois groups* over complex, p -adic numbers and finite fields.

Algebraically these are modules over group rings (*Galois modules*).

That is why the theory of modules, the basic object of A. V. Mikhalev study, is a fundamental object in several branches of mathematics.

2. MODULES AND COMPUTER ALGEBRA

Methods of Computer algebra allow us to work concretely with modules and provide a stimulation for new results and conjectures.

The influence of *A.V. Mikhalev, E.V. Pankratiev* and their Laboratory of Computer Methods in the period of computerization in 1987-1989 allowed Prof. *Yuri Ivanovich Manin* to propose his famous Conjecture of Linear growth in the Diophantine Geometry on the basis of computations. This links this work with Algebraic Geometry.

3. THE SCHOOL OF A. V. MIKHALEV AND THE ALGEBRAIC SCHOOL IN MOSCOW

Thanks to the Algebraic School of Moscow State University, new perspectives and paradigms were opened such as derived categories, geometric representation theory, and new cohomology theories (motivic cohomology of *V.A. Voevodsky*, prismatic cohomology of *P. Scholze, V. Berkovich* geometry etc.).

Undoubtedly, numerous new theories were created under the influence of A.V. Mikhalev's School, and whole generations of scientists were formed under his leadership.

4. INTERESTS OF A. V. MIKHALEV RELATED TO THE GRADED STRUCTURES IN ALGEBRA, NON-ARCHIMEDIAN ANALYSIS, AND THE WORK OF MARC KRASNER AND HIS FOLLOWERS: Moscow – Sarajevo – Paris – Grenoble.

We both had a privilege to be among numerous collaborators of *A.V. Mikhalev* all over the world. One of the authors of this text, *Mirjana Vuković*, a former student of *Marc Krasner*, met A.V. Mikhalev during a stay at *Lomonosov – Moscow State University* (1975-76) and in 2000-2001 completed an important scientific visit to the *Fourier Institute* (France) supported by a regional cooperation program Rhôn-Alpes (TEMPRA-PECO) together with professors *A.N. Andrianov* (from Sankt-Petersburg, Russia) and *Siegfried Boecherer* (from Mannheim, Germany).

Finally, what else can we say about Alexander Vasilevich Mikhalev? On the one hand, a great scientist, a great man, a real giant, and on the other hand, if one looks at him closely, his whole face radiated kindness, his blue eyes smiled at everyone and looked at everyone with gentleness and kindness, and understanding radiated from them. In short, he was a very good man. And that's why we'll just say, may he rest in peace, his memory will live on with everyone who knew him.

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