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CONSTRUCTION OF A SUBSPACE OF THE SPECTRUM OF L FROM THE L-SLICE FOR A LOCALE L

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ABSTRACT. The notion of compactness in the L-slice (σ,J) of a locale L is introduced and we show that L-slice compactness is stronger than topological compactness and localic compactness. A subspace Y, of the spectrum $Sp(L)$ of the locale L, is constructed using filters $F_x = \{a \in L : \sigma(a,x) = x\}$ for compact elements $x \in (\sigma,J)$ and the compactness of the subspace Y is characterised using the existence of a maximal compact element in the L-slice (σ,J).

1. INTRODUCTION

A frame (locale) is a complete lattice satisfying the infinite distributive property $a \sqcap \bigsqcup B = \bigsqcup\{a \sqcap b; b \in B\}$ for all $a \in L$ and $B \subseteq L$. Locales (frames) resemble the lattice of open subsets of a topological space, and thus many topological questions can be solved using locale theory. We can see a locale as a kind of space, more general than the classical one, allowing us to see topological phenomena in a new perspective. Also the theory of locales has a connection with domain theory [16] [7], continuous lattices [7] [3], logic [17] [5] and topos theory [5].
The action of residuated structures on posets has been studied by many authors, especially in connection with mathematical logic. Such structures are involved in the development of several interesting theories including image processing [14]. The concept of action of a locale L on a join semilattice J is introduced in [15]. The L-slice though algebraic in nature adopts topological properties such as compactness of L through the action σ. In this paper we prove the following results:
i. For each $x \in (\sigma,J)$, $F_x = \{a \in L; \sigma(a,x) = x\}$ is a filter in the locale L, which is proper for $x \neq 0_J$. We prove that F_x is a prime filter, if x is a join irreducible element of (σ,J).
ii. A stronger notion of a compact element in an L-slice (σ,J) is introduced and it is characterized using the filter F_x. A subspace of $Sp(L)$ is constructed using the filters F_x.

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iii. If $Y = \{F_x : x \text{ is join irreducible and compact in } (\sigma, J)\} \in \Omega(Sp(L))$ for a spatial locale L, then Y corresponds to a sublocale of L. The method in this paper suggests a way of obtaining a sublocale of the locale L from the L-slice of a locale L.

The locale L can be viewed as an L-slice in which case the action σ is the meet.

Notation

- $\sqsubseteq$: Partial order relation in the locale
- $\sqcap$: Meet in the locale
- $\sqcup$: Join in the locale
- $\leq$: Partial order relation in the join semilattice
- $\wedge$: Meet in the join semilattice
- $\vee$: Join in the join semilattice
- $\Omega(X)$: The Open set lattice of the topological space X
- $\rightarrow$: Heyting operation $a \rightarrow b$ in a bounded lattice L is the greatest $c \in L$ such that $a \wedge c \leq b$.

2. Preliminaries

Definition 2.1. *[6] A lattice $(L, \leq)$ is said to be a lattice with a relative pseudo-complement if for any elements $a, b \in L$ the set $\{d \in L : a \wedge d \leq b\}$ has a greatest element, which is denoted by $a \rightarrow b$.*

Definition 2.2. *[6] A lattice L with a relative pseudocomplement $\rightarrow$ is said to be a Heyting lattice if L has a least element 0. The element $a \rightarrow 0$ of a Heyting lattice $(L, \leq)$ is called the pseudocomplement of a and is denoted by $\neg a$.*

Proposition 2.1. *[6] A Heyting lattice L is a bounded distributive lattice.*

Definition 2.3. *An element a in a lattice L is said to be join irreducible if and only if a is not a bottom element, and, whenever $a = b \vee c$, then $a = b$ or $a = c$.*

Definition 2.4. *Let L be a poset. We say that x is way below y, $(x \ll y)$ if and only if for all directed subsets $D \subseteq L$ for which supD exists, the relation $y \leq supD$ always implies the existence of a $d \in D$ with $x \leq d$.*

Definition 2.5. *An element satisfying $x \ll x$ is said to be compact or isolated from below.*

Definition 2.6. *[13] A frame is a complete lattice L satisfying the infinite distributivity law $a \sqcap \bigsqcup B = \bigsqcup \{a \sqcap b; b \in B\}$ for all $a \in L$ and $B \subseteq L$.*

Example 2.1. *[13] i.The lattice of open subsets of a topological space.*
ii.Every complete Heyting lattice.
iii. Every complete Boolean algebra.

Definition 2.7. *[13] A map $f : L \to M$ between frames L, M preserving all finite meets (including the top 1) and all joins (including the bottom 0) is called a frame homomorphism. A bijective frame homomorphism is called a frame isomorphism.*

The category of frames is denoted by **Frm**. The opposite of the category **Frm** is the category **Loc** of locales.

Definition 2.8. *[10] A subset I of a locale L is said to be an ideal if*
i. I is a sub-join-semilattice of L; that is $0_L \in I$ and $a \in I, b \in I$ implies $a \sqcup b \in I$; and
ii. I is a lower set; that is $a \in I$ and $b \sqsubseteq a$ imply $b \in I$.

Definition 2.9. *[13] A subset F of locale L is said to be a filter if*
i. F is a sub-meet-semilattice of L; that is $1_L \in F$ and $a \in F$, $b \in F$ imply $a \sqcap b \in F$.
ii. F is an upper set; that is $a \in F$ and $a \sqsubseteq b$ imply $b \in F$.

Definition 2.10. *[13] A filter F is proper if $F \neq L$, that is if $0_L \notin F$. A proper filter F in a locale L is prime if $a_1 \sqcup a_2 \in F$ implies that $a_1 \in F$ or $a_2 \in F$*

Definition 2.11. *[13] A proper filter F in a locale L is a completely prime filter if for any indexing set J and $a_i \in L$, $i \in J$, $\bigsqcup a_i \in F \Rightarrow \exists i \in J$ so that $a_i \in F$. Completely prime filters are called c.p filters.*

For an element a of a locale L, set $\Sigma_a = \{F \subseteq L; F \neq \phi, F \ is \ a \ c.p \ filter; a \in F\}$. We can easily check that $\Sigma_0 = \phi$, $\Sigma_{\bigsqcup a_i} = \bigcup \Sigma_{a_i}$, $\Sigma_{a \sqcap b} = \Sigma_a \cap \Sigma_b$ and $\Sigma_1 = \{all \ c.p \ filters\}$. Thus the collection of all completely prime filters together with the collection $\{\Sigma_a : a \in L\}$ forms a topological space and it is called the spectrum Sp(L) associated with the locale L. The topology on Sp(L) is $\Omega(Sp(L)) = \{\Sigma_a : a \in L\}$.

Definition 2.12. *A locale L is said to be spatial if it is isomorphic to $\Omega(X)$ for some topological space X.*

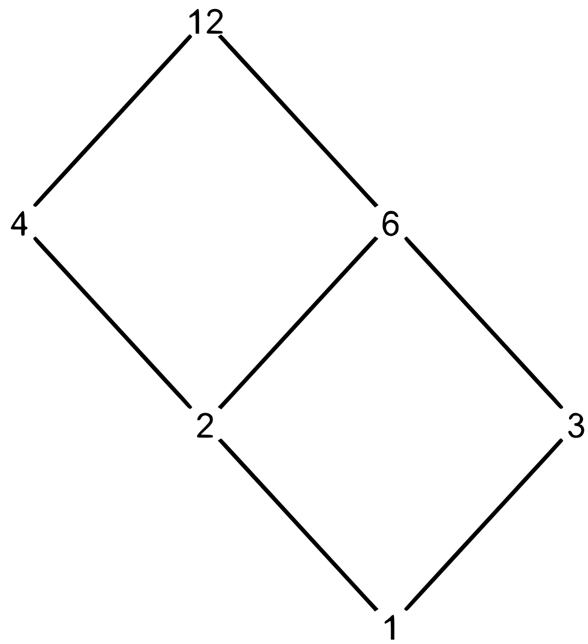
Definition 2.13. *A subset of a frame L which is closed under the same finite meets and arbitrary joins in the frame is called a subframe. That is a subframe is itself a frame under the induced order of L.*

The concept of sublocale is something different, corresponding to quotient frames.

Definition 2.14. *[13] Let L be a locale. A subset $S \subseteq L$ is a sublocale of L if*
i. S is closed under all meets, and
ii. For every $s \in S$ and every $x \in L$, $x \to s \in S$, where $x \to s$ is the greatest $c \in L$ such that $x \wedge c \leq s$.

A sublocale is always nonempty, since $1 = \bigsqcap \phi \in S$. The least sublocale $\{1\}$ will be denoted by **0**.

Example 2.2. *[13] Consider the locale L given below and let* $S = \{4, 12\}$. *S is*



closed under all meets.
$1 \to 4 = 2 \to 4 = 4 \to 4 = 12 \in S$
$3 \to 4 = 6 \to 4 = 12 \to 4 = 4 \in S$
$1 \to 12 = 2 \to 12 = 3 \to 12 = 4 \to 12 = 6 \to 12 = 12 \in S$
Hence S is a sublocale of L

Definition 2.15. *[13] Let L be a locale and* $a \in L$. *The open sublocale associated with a is defined by* $o(a) = \{a \to x, x \in L\}$.

Definition 2.16. *[13] Let L be a locale and* $a \in L$. *The closed sublocale associated with a is defined by* $c(a) = \{x \in L, a \sqsubseteq x\} = \uparrow a$.

Definition 2.17. *[13] A cover of a locale L is a subset* $A \subseteq L$ *such that* $\bigsqcup A = 1$. *A subcover of a cover A is a subset* $B \subseteq A$ *such that* $\bigsqcup B = 1$. *A locale is said to be compact if each cover has a finite subcover.*

Definition 2.18. *[15] Let L be a locale and J be a join semilattice with bottom element* 0_J. *By the "action of L on J"we mean a function* $\sigma : L \times J \to J$ *such that the following conditions are satisfied.*
i. $\sigma(a, x_1 \vee x_2) = \sigma(a, x_1) \vee \sigma(a, x_2)$ *for all* $a \in L, x_1, x_2 \in J$.
ii. $\sigma(a, 0_J) = 0_J$ *for all* $a \in L$.
iii. $\sigma(a \sqcap b, x) = \sigma(a, \sigma(b, x)) = \sigma(b, \sigma(a, x))$ *for all* $a, b \in L, x \in J$.

iv. $\sigma(1_L,x) = x$ *and* $\sigma(0_L,x) = 0_J$ *for all* $x \in J$.
v. $\sigma(a \sqcup b,x) = \sigma(a,x) \vee \sigma(b,x)$ *for* $a,b \in L, x \in J$.

If σ *is an action of the locale L on a join semilattice J, then we call* (σ,J) *an L-slice.*

Remark 2.1. We use the notation (σ,J) for an L-slice in order to emphasize that the join semilattice J is equipped with the action of a locale.

Example 2.3. *[15] 1. Every locale L is an L-slice under the action* $\sqcap$.
2.Let I be an ideal of a locale L. Define $\sqcap : L \times I \to I$ *by* $\sqcap(a,i) = a \sqcap i$. *Then* $\sqcap$ *is an action of L on join semilattice I. Hence every ideal of a locale is an L-slice.*
3. Let the locale L have the property that $a \sqcap b = 0$ *implies that either* $a = 0$ *or* $b = 0$. *J be any join semilattice with bottom element. Define* $\sigma : L \times J \to J$ *by* $\sigma(a,j) = j \ \ \forall a \neq 0$ *and* $\sigma(0_L,j) = 0_J$. *Then* σ *is an action of L on J and* (σ,J) *is an L-slice.*

Definition 2.19. *[15] Let* (σ,J) *be an L-slice of a locale L. A subjoin semilattice* J' *of J is said to be L-subslice of J if* J' *is closed under the action by elements of L.*

Definition 2.20. *[15] Let* (σ,J), (μ,K) *be L-slices of a locale L. A map* $f : (\sigma,J) \to (\mu,K)$ *is said to be an L-slice homomorphism if*
i. $f(x_1 \vee x_2) = f(x_1) \vee f(x_2)$ *for all* $x_1,x_2 \in (\sigma,J)$.
ii. $f(\sigma(a,x)) = \mu(a,f(x))$ *for all* $a \in L$ *and all* $x \in (\sigma,J)$.

Proposition 2.2. *[15] Let* (σ,J) *be an L-slice of a locale L. For each* $x \in J, \sigma_x : (\sqcap,L) \to (\sigma,J)$ *defined by* $\sigma_x(a) = \sigma(a,x)$ *is an L-slice homomorphism.*

3. Filters in L with respect to the slice (σ,J)

Let (σ,J) be an L-slice of a locale L and let $x \in J$. We look into various properties of the collection $F_x = \{a \in L; \sigma_x(a) = x\}$.

Proposition 3.1. *Let* (σ,J) *be an L-slice of a locale L. For each* $x \in J$, *let* $F_x = \{a \in L; \sigma(a,x) = x\}$. *Then* F_x *is a filter in L.*

Proof. By Definition 2.18(iv), $1 \in F_x$. Let $a,b \in F_x$. Then $\sigma(a,x) = \sigma(b,x) = x$. $\sigma(a \sqcap b,x) = \sigma(a,\sigma(b,x)) = \sigma(a,x) = x$. Hence $a \sqcap b \in F_x$. Let $a \in F_x$ and $c \in L$ such that $a \sqsubseteq c$. $\sigma(a,x) = \sigma(a \sqcap c,x) = \sigma(c \sqcap a,x) = \sigma(c,\sigma(a,x)) = \sigma(c,x)$. Thus F_x is a filter in L. □

Proposition 3.2. *Let* (σ,J) *be an L-slice and* $x \leq y \in J$. *Then*
i. $x \leq \sigma(a,y)$ *for all* $a \in F_x$.
ii. $\sigma(b,x) \leq y$ *for all* $b \in F_y$.

Proof. i. Let $x \leq y \in J$ and $a \in F_x$. Then $\sigma(a,x) = x$. $\sigma(a,y) = \sigma(a,x \vee y) = \sigma(a,x) \vee \sigma(a,y) = x \vee \sigma(a,y)$. Hence $x \leq \sigma(a,y)$ for all $a \in F_x$.
ii. Let $b \in F_y$. Then $\sigma(b,y) = y$. $\sigma(b,y) = \sigma(b,x \vee y) = \sigma(b,x) \vee \sigma(b,y) = \sigma(b,x) \vee y$. Hence $\sigma(b,x) \leq y$ for all $b \in F_y$. □

Proposition 3.3. *The filter F_x is proper for $x \neq 0_J$.*

Proof. Suppose $x \neq 0_J$. If $0_L \in F_x$, then $\sigma(0_L, x) = x$, which implies $0_J = \sigma(0_L, x) = x$. Hence if $x \neq 0_J$, $0_L \notin F_x$ and so F_x is proper. □

Proposition 3.4. *Consider the L-slice $(\sqcap, L)$. Then for each $b \in L, F_b$ is a closed sublocale of L..*

Proof. $F_b = \{a \in L : \sqcap(a,b) = b\} = \{a \in L : a \sqcap b = b\} = \{a \in L : a \sqsupseteq b\}$ $= \uparrow b$. Hence F_b is a closed sublocale of L. □

Proposition 3.5. *Let (σ, J) be an L-slice of a locale and $x \in J$ be a join-irreducible element, then F_x is a prime filter in L.*

Proof. By proposition 3.1, F_x is a filter in L. Let $a \sqcup b \in F_x$. Then $\sigma(a \sqcup b, x) = x$. That is $\sigma(a,x) \vee \sigma(b,x) = x$. Since x is join-irreducible, $\sigma(a,x) = x$ or $\sigma(b,x) = x$. Hence either $a \in F_x$ or $b \in F_x$. Thus F_x is a prime filter in L. □

Definition 3.1. *An element $x \in J$ is said to be a compact element of the L-slice (σ, J), if for any collection $\{a_\alpha\}$ of L whenever $\sigma(\sqcup a_\alpha, x) = x$, there exists a finite sub collection $\{a_1, a_2, ...a_n\}$ of $\{a_\alpha\}$ such that $\sigma(a_1,x) \vee \sigma(a_2,x) \vee ... \vee \sigma(a_n,x) = x$. A slice (σ, J) is compact if each element $x \in J$ is compact.*

From the definition of compact L-slice, it is clear that every L-subslice of a compact L-slice is compact.

Example 3.1. *Let (σ, J) be any L-slice. Then 0_J is a compact element.*

Proposition 3.6. *Let L be a locale. If the L-slice $(\sqcap, L)$ is compact, then the locale L is compact.*

Proof. Suppose the L-slice $(\sqcap, L)$is compact and let $\{a_\alpha\} \in L$ such that $\sqcup a_\alpha = 1$. Then for any $b \in (\sqcap, L), (\sqcup a_\alpha) \sqcap b = 1 \sqcap b = b$. Since $(\sqcap, L)$ is compact, there exist a finite sub collection $\{a_1, a_2, ...a_n\}$ of $\{a_\alpha\}$ such that $(a_1 \sqcup a_2 \sqcup ... \sqcup a_n) \sqcap b = b$. In particular this is true for $b = 1$. Hence $(a_1 \sqcup a_2 \sqcup ... \sqcup a_n) \sqcap 1 = 1$. Then $(a_1 \sqcup a_2 \sqcup ... \sqcup a_n) = 1$ and hence the locale L is compact. □

The above proposition shows that the notion of compactness in the L-slice is stronger than the topological compactness and compactness in locale.

Proposition 3.7. *Let L be a compact locale and (σ, J) be an L-slice. Let $x \in J$ be such that σ_x is one one. Then x is a compact element of the L-slice (σ, J).*

Proof. Let L be a compact locale and $x \in J$. Suppose $\sigma(\sqcup a_\alpha, x) = x$. That is $\sigma_x(\sqcup a_\alpha) = x = \sigma_x(1)$. Since $\sigma_x : L \to J$ is one one, $\sqcup a_\alpha = 1$. Since L is a compact locale, there exist a finite sub collection $a_1, a_2, ...a_n$ of $\{a_\alpha\}$ such that $a_1 \sqcup a_2 \sqcup ... \sqcup a_n = 1$. Then $\sigma(a_1 \sqcup a_2 \sqcup ... \sqcup a_n, x) = \sigma(1, x) = x$. Hence x is a compact element of the L-slice (σ, J). □

Corollary 3.1. *Let L be a compact locale and* (σ, J) *be an L-slice. If* σ_x *is one-one for every* $x \in J$*, then* (σ, J) *is a compact L-slice.*

Corollary 3.2. *Let L be a compact locale and let* $\sqcap_x : L \to L$ *be one-one for every* $x \in L$*. Then the L-slice* $(\sqcap, L)$ *is compact.*

Proposition 3.8. *Let* $(\sigma, J), (\mu, K)$ *be L-slices of a locale L and* $f : (\sigma, J) \to (\mu, K)$ *be a one-one L-slice homomorphism. If x is a compact element of the L-slice* (σ, J)*, then* $f(x)$ *is a compact element of the L-slice* (μ, K)*.*

Proof. Let $f : (\sigma, J) \to (\mu, K)$ be a one-one L-slice homomorphism and let x be a compact element of the L-slice (σ, J). Let $\{a_\alpha\} \in L$ such that $\mu(\sqcup a_\alpha, f(x)) = f(x)$. Since f is a one-one L-slice homomorphism, $\sigma(\sqcup a_\alpha, x) = x$. Then by compactness of the element $x \in (\sigma, J)$, there exist a finite sub collection $a_1, a_2, ...a_n$ of $\{a_\alpha\}$ such that $\sigma(a_1 \sqcup a_2 \sqcup .... \sqcup a_n, x) = x$. Then we have $\mu(a_1 \sqcup a_2 \sqcup .... \sqcup a_n, f(x)) = f(\sigma(a_1 \sqcup a_2 \sqcup .... \sqcup a_n, x)) = f(x)$. Hence $f(x)$ is a compact element of the L-slice (μ, K). □

Definition 3.2. *A proper filter F in a locale L is a partially completely prime filter if for any indexing set I and* $a_i \in L, i \in I$, $\bigsqcup a_i \in F \Rightarrow \exists a_1, a_2 ....a_n$ *such that* $a_1 \sqcup a_2 \sqcup ... \sqcup a_n \in F$*.*

Proposition 3.9. *Let* (σ, J) *be an L-slice of a locale L and* $x \in J$*. Then x is a compact element of the slice* (σ, J) *if and only if the filter* F_x *is partially completely prime.*

Proof. Suppose x is a compact element of the L-slice (σ, J). Let $\sqcup a_\alpha \in F_x$. Then we have $\sigma(\sqcup a_\alpha, x) = x$. Since x is a compact element, there is a finite collection $\{a_1, a_2, ...a_n\}$ of $\{a_\alpha\}$ such that $\sigma(a_1, x) \vee \sigma(a_2, x) \vee ... \vee \sigma(a_n, x) = x$. That is $\sigma(a_1 \sqcup a_2 \sqcup ... \sqcup a_n, x) = x$. So $a_1 \sqcup a_2 \sqcup ... \sqcup a_n \in F_x$ and hence F_x is a partially completely prime filter. The converse is simple. □

Proposition 3.10. *Let* $x \in J$ *be a join-irreducible compact element of* (σ, J)*. Then* F_x *is a completely prime filter.*

Proof. Let $x \in J$ be a join-irreducible compact element of (σ, J) and let $\sqcup a_\alpha \in F_x$. Since x is a compact element, by proposition 3.9, F_x is a partially completely prime filter. Hence there is $a_1, a_2, ...a_n \in \{a_\alpha\}$ such that $a_1 \sqcup a_2 \sqcup ... \sqcup a_n \in F_x$. Since x is join-irreducible element of (σ, J), there is some a_i such that $a_i \in F_x$. Hence F_x is a completely prime filter. □

Definition 3.3. *A compact element x of an L-slice* (σ, J) *of a locale L is said to be a maximal compact element if the filter* F_x *associated with* $x \in J$ *has the property that* $F_x \subseteq F_y$ *for all compact elements* $y \in (\sigma, J)$*.*

Proposition 3.11. *If the collection of all compact elements of a locale forms a chain with maximal element x, then x is a maximal compact element of the L-slice* $(\sqcap, L)$*.*

Proof. By Proposition 3.4, $F_x = \uparrow x$. Let $y \in L$ be any other compact element of the L-slice $(\sqcap, L)$. Since $y \leq x, F_x = \uparrow x \subseteq \uparrow y = F_y$. Hence x is a maximal compact element of the L-slice $(\sqcap, L)$. □

4. Construction- The Subspace of Sp(L)

Proposition 4.1. *Let* $Y = \{F_x : x \text{ is join irreducible and compact in } (\sigma, J)\}$. *Then* $(Y, \Omega(Sp(L))/Y)$ *is a topological space, where* $\Omega(Sp(L))/Y$ *denotes the subspace topology on Y.*

Proof. If $x \in J$ is a join-irreducible compact element, then by proposition 3.10, F_x is a completely prime filter and so $Y \subseteq Sp(L)$. Then $\Omega(Sp(L))/Y$ is the subspace topology on Y and $(Y, \Omega(Sp(L))/Y)$ is a topological space. □

Example 4.1. *i. Let* $(\sqcap, L)$ *be an L-slice. By Proposition 3.4, we have* $F_b = \uparrow b$*, a principal filter of L. Then* $Y = \{\uparrow b : b \in (\sqcap, L) is\ a\ join-irreducible\ compact\ element\}$. *But* $b \in L$ *is join-irreducible and a compact element if and only* $\uparrow b$ *is a completely prime filter in the locale L. Hence* $Y = Sp(L)$.

ii. Let L be a locale and let I be an ideal of L.Consider the L-slice $(\sqcap, I)$. *As in the case of the above example,* $Y = \{\uparrow b : \uparrow b \text{ is a completely prime filter in } L, b \in (\sqcap, I)\} \subsetneq Sp(L)$. *Hence* $(Y, \Omega(Sp(L))/Y)$ *is a proper subspace of* $Sp(L)$.

iii. Let the L-slice (σ, J) *be the L-slice of example 2.3(ii). For* $x \in J$, $F_x = L \backslash \{0_L\}$. *Hence* $Y = L \backslash \{0_L\}$. *Let* $b \in L$ *be a minimal element of L. Then* $Y = L \backslash \{0_L\} = \Sigma_b$. *So Y is an open subset of* $Sp(L)$ *and* $\Omega(Sp(L))/Y$ *is isomorphic to* **2**.

The subspace $(Y, \Omega(Sp(L))/Y)$ depends on the L-slice (σ, J). If Y is an open set in $Sp(L)$, then $\Omega(Sp(L))/Y$ is a sublocale of the locale $\Omega(Sp(L))$ and hence a sublocale of L. If $Y = Sp(L)$, the points of L are completely determined by the L-slice (σ, J).

Proposition 4.2. *If the L-slice* (σ, J) *of a locale L has a maximal compact irreducible element z, then* $(Y = \{F_x : x \text{ is join irreducible and compact in } (\sigma, J)\}$, $\Omega(Sp(L))/Y)$ *is a compact subspace of the spectrum* $Sp(L)$ *of the locale L.*

Proof. Let $\{\Sigma_{a_\alpha} : \alpha \in I\}$ be an open cover for Y. Then $Y \subseteq \bigcup \Sigma_{a_\alpha} = \Sigma_{\sqcup a_\alpha}$. Since $F_z \in Y, F_z \in \Sigma_{\sqcup a_\alpha}$ or $\sqcup a_\alpha \in F_z$. Since F_z is a completely prime filter, there is some $\beta \in I$ such that $a_\beta \in F_z$. Then $F_z \in \Sigma_{a_\beta}$. Also for any $F_x \in Y$, we have $a_\beta \in F_z \subseteq F_x$. Hence $Y \subseteq \Sigma_{a_\beta}$ and so Y is compact. □

References

[1] R. P. Agarwal, M. Meehan and D. O'Regan, *Fixed Point Theory and Applications*, Cambridge Univ. Press, United Kingdom, 2001.

[2] M.F.Atiyah,I.G.Macdonald,Introduction to commutative algebra ,Addison-Wesley Publishing Company.

[3] B. Banaschewski, Coherent frames, in: Continuous Lattices, Springer Lecture Notes in Math. 871, (1981) pp. 1-11.
[4] G.Birkhoff,Lattice Theory,American Mathematical Society.
[5] F. Borceux, Handbook of Categorical Algebra, Encyclopedia of Mathematics and its Applications 52,Cambridge University Press, Cambridge, 1994.
[6] L.Esaka, Heyting Algebras Duality Theory, Springer 2019, English translation of the original 1985 text book.
[7] G Gierz, K.H Hofmann, K Keimel, J.D Lawson, M Mislove, D.S Scott, *Continuous lattices and Domains*, Cambridge University press,2003.
[8] George Gratzer,General lattice theory,Birkhauser,2003.
[9] P.T.Johnstone,The point of pointless topology,Bulletin of American Mathematical society,1983.
[10] P.T.Johnstone,StoneSpaces,CambridgeUniversityPress,1982.
[11] H.Matsumara,Commutative algebra,Addison Wesley Longman 1970.
[12] C.Musli,Introduction to Rings and Modules,Narosa Publishing House,1994.
[13] J.Picado,Pultr,Frames and locales:Topology without points,Front.Math.,Springer,Basel,2012.
[14] Russo C, Quantale modules, with applications to logic and image processing, Ph.D Thesis, University of Salerno-Italy 2007.
[15] Sabna K.S, Mangalambal N.R,Fixed points with respect to L-slice homomorphism σ_a, Archivum Mathematicum, ()2019) pp.43-53.
[16] G. Sambin, Formal topology and domains, Electronic Notes in Theoretical Computer Science 2000.
[17] S. Vickers, Topology via Logic, in: Cambridge Tracts in Theoretical Computer Science, Vol. 5, Cambridge University Press, Cambridge, 1985.

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