

SUBTRACTIVE EXTENSION OF IDEALS IN SEMIRINGS

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ABSTRACT. In this paper, we (1) obtain the k -closure of ideals and a characterization of subtractive extension of ideals in the semiring \mathbb{Z}_0^+ ; (2) introduce the concept of closure of an ideal A of a semiring R with respect to an ideal I of R and prove the set of all subtractive extensions of an ideal I of a semiring R is a complete lattice; (3) show that a subtractive extension P of a Q -ideal I in a semiring R is a semiprime ideal if and only if $P/I_{(Q \cap P)}$ is a semiprime ideal in the quotient semiring $R/I_{(Q)}$.

1. INTRODUCTION

The theory of semirings was first developed by H.S. Vandiver and he has obtained important results for these objects. Semirings constitute a fairly natural generalization of rings, with board applications in the mathematical foundation of computer science. Also, semiring theory has many applications to other branches. For example, automata theory, optimization theory, algebra of formal processes, combinatorial optimization, Bayesian networks and belief propagation.

A *semiring* is a system consisting of a set R together with two binary operations on R called *addition* and *multiplication* (denoted in the usual manner) such that

- (1) R together with addition is a (commutative) semigroup,
- (2) R together with multiplication is a semigroup,
- (3) there exists $0 \in R$ such that $x + 0 = x = 0 + x$, $x \cdot 0 = 0 = 0 \cdot x$ for each $x \in R$
- (4) $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$ for each $a, b, c \in R$.

We suppose that all semirings in this paper are commutative.

Let \mathbb{B} be any Boolean algebra. Then, (\mathbb{B}, \cup, \cap) is a semiring. Let I be the real interval $[0, 1]$. Then, (I, \max, \min) is a semiring.

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The concept of ideal, finitely generated ideal, principal ideal, semiprime ideal in a commutative semiring with identity 1 can be defined as for commutative rings with identity 1. Denote the sets of all non-negative, and positive integers respectively by \mathbb{Z}_0^+ , and \mathbb{N} . \mathbb{Z}_0^+ is a semiring under usual addition and multiplication of non-negative integers but it is not a ring. For $a_1, a_2, \dots, a_n \in \mathbb{Z}_0^+$, we denote (i) $\langle a_1, a_2, \dots, a_n \rangle$ = the ideal generated by a_1, a_2, \dots, a_n in the semiring \mathbb{Z}_0^+ ; (ii) (a_1, a_2, \dots, a_n) = g.c.d. of a_1, a_2, \dots, a_n . An ideal I of a semiring R is called a *subtractive ideal* (= *k-ideal*) if $a, a+b \in I, b \in R$, then $b \in I$. An ideal I of a semiring R is called a *Q-ideal* (as introduced by P. J. Allen [2]) if there exists a subset Q of R such that

- (1) $R = \cup\{q + I : q \in Q\}$.
- (2) if $q_1, q_2 \in Q$, then $(q_1 + I) \cap (q_2 + I) \neq \emptyset \Leftrightarrow q_1 = q_2$.

Let I be a Q -ideal of a commutative semiring R with identity element. Then, $R/I_{(Q)} = \{q+I : q \in Q\}$ forms a semiring under the binary operations “ \oplus ” and “ \odot ” defined as follows:

$$(q_1 + I) \oplus (q_2 + I) = q' + I$$

where $q' \in Q$ is a unique element such that $q_1 + q_2 + I \subseteq q' + I$.

$$(q_1 + I) \odot (q_2 + I) = q'' + I$$

where $q'' \in Q$ is a unique element such that $q_1 q_2 + I \subseteq q'' + I$. This semiring $R/I_{(Q)}$ is called a quotient semiring of R by I . If $q_0 \in Q$ is the unique element such that $q_0 + I = I$, then $q_0 + I$ is the zero element of $R/I_{(Q)}$ [7, Proposition 8.21].

If A is an ideal of a semiring R , then $\bar{A} = \{x \in R : x + y \in A \text{ for some } y \in A\}$, is called the *k-closure* of A [6].

The ideal theory in the quotient of commutative semirings was studied by Atani [1]. In [4], Chaudhari and Bonde introduced the concept of the subtractive extension of an ideal in semirings. In Section 2, we obtain *k-closure* of ideals and a characterization of subtractive extension of ideals in the semiring $(\mathbb{Z}_0^+, +, \cdot)$. In Section 3, we introduce the concept of closure of an ideal A of a semiring R with respect to an ideal I of R and prove the set of all subtractive extensions of an ideal I of a semiring R is a complete lattice. Also we show that a subtractive extension P of a Q -ideal I in a semiring R is a semiprime ideal if and only if $P/I_{(Q \cap P)}$ is a semiprime ideal in the quotient semiring $R/I_{(Q)}$.

The following results will be used to prove our results.

Lemma 1.1 ([3]). *Let $I = \langle a_1, a_2, \dots, a_n \rangle \subseteq \mathbb{Z}_0^+$. If $(a_1, a_2, \dots, a_n) = d$, then there exists a smallest $r \in \mathbb{Z}_0^+$ such that $dk \in I$ for all $k \geq r$.*

Theorem 1.2. ([8, Theorem 4]) *Every ideal of \mathbb{Z}_0^+ is finitely generated.*

Theorem 1.3. ([5, Proposition 2.19]) *An ideal I of \mathbb{Z}_0^+ is subtractive if and only if I is a principal ideal.*

2. k -CLOSURE AND SUBTRACTIVE EXTENSIONS OF IDEALS IN THE SEMIRING \mathbb{Z}_0^+

In this section, we obtain the k -closure of ideals and a characterization of the subtractive extension of ideals in the semiring $(\mathbb{Z}_0^+, +, \cdot)$.

Lemma 2.1. *Let $I = \langle b_1, b_2, \dots, b_m \rangle$ be a non-zero ideal of $R = (\mathbb{Z}_0^+, +, \cdot)$ and $d = (b_1, b_2, \dots, b_m)$. Then, $\bar{I} = \langle d \rangle$.*

Proof. Let $x \in \bar{I}$. Then, $x + i \in I$ for some $i \in I$. Since $d \mid b_j$ for all j , $I = \langle b_1, b_2, \dots, b_m \rangle \subseteq \langle d \rangle$. Hence, $x + i \in \langle d \rangle$ where $i \in I \subseteq \langle d \rangle$. By Theorem 1.3, $x \in \langle d \rangle$. Now, $\bar{I} \subseteq \langle d \rangle$. For the other inclusion, by Lemma 1.1, there exists $r \in \mathbb{Z}_0^+$ such that $dk \in I$ for all $k \geq r$. Then, $dr + d = d(r + 1)$, $dr \in I$. Hence, $d \in \bar{I}$. So $\langle d \rangle \subseteq \bar{I}$. Therefore, $\bar{I} = \langle d \rangle$. \square

Let I be an ideal of a semiring R . An ideal A of R with $I \subseteq A$ is said to be a subtractive extension of I if $x \in I$, $x + y \in A$, $y \in R$, then $y \in A$ [4]. Clearly, every subtractive ideal A of a semiring R containing an ideal I of R is a subtractive extension of I . Also every ideal of a semiring R is a subtractive extension of $\{0\}$.

Example 2.2. Let $I = \{0\}$, $A = \langle 2, 3 \rangle$ be ideals in the semiring $R = (\mathbb{Z}_0^+, +, \cdot)$. Then, A is a subtractive extension of I but by Theorem 1.3. it is not a subtractive ideal.

By Theorem 1.2, every ideal of $(\mathbb{Z}_0^+, +, \cdot)$ is finitely generated. Now, the following theorem gives a characterization of subtractive extensions of non-zero ideals in the semiring $(\mathbb{Z}_0^+, +, \cdot)$:

Theorem 2.3. *Let $I = \langle b_1, b_2, \dots, b_m \rangle$ be a non-zero ideal of $R = (\mathbb{Z}_0^+, +, \cdot)$ and $d = (b_1, b_2, \dots, b_m)$. Then, an ideal A of R is a subtractive extension of I if and only if $A = \langle a \rangle$ where $a \mid d$.*

Proof. Let A be a subtractive extension of I . Suppose that A is not a principal ideal. Then, by Theorem 1.2, $A = \langle a_1, a_2, \dots, a_n \rangle$ where $1 < a_1 < a_2 < \dots < a_n$, $a_i \nmid a_j$ for all $i < j$, $j = 2, 3, \dots, n$, $n \geq 2$. Let $d' = (a_1, a_2, \dots, a_n)$. By Lemma 1.1, there exist $r_1, r_2 \in \mathbb{N}$ such that $dk \in I$ and $d's \in A$ for all $k \geq r_1$, $s \geq r_2$. Hence, $dk \in I$ and $d'k \in A$ for all $k \geq r$ where $r = \max\{r_1, r_2\} \dots (1)$. Since $d, d' > 0$, $d'r, dr + 1 \geq r$. So by (1), $dd'r \in I$ and $dd'r + d' = d'(dr + 1) \in A$. Since A is a subtractive extension of I , $d' \in A$. Hence, $d' = a_1$. So $a_1 \mid a_2$ a contradiction. Now, $A = \langle a \rangle$ for

some $a \in \mathbb{Z}_0^+$. Since $I \subseteq A$, $a \mid d$. Conversely, suppose that $A = \langle a \rangle$ where $a \mid d$. Clearly, $I \subseteq A$. By Theorem 1.3, A is a subtractive ideal of R and hence A is a subtractive extension of I . \square

The following example shows that the sum (union) of two subtractive extensions of I need not be subtractive extension of I .

Example 2.4. Let $I = \langle 12, 18 \rangle$, $A = \langle 2 \rangle$ and $B = \langle 3 \rangle$ be ideals in the semiring \mathbb{Z}_0^+ . By Theorem 2.3, A, B are subtractive extensions of I , but $A + B = \{0, 2, 3, 4, 5, 6, \dots\} = \langle 2, 3 \rangle$, is not a subtractive extension of I . An inspection will show that $A \cup B$ is not an ideal of \mathbb{Z}_0^+ and hence $A \cup B$ is not a subtractive extension of I .

Denote $\mathbb{Z}_0^+ \times \mathbb{Z}_0^+ = (\mathbb{Z}_0^+, +, \cdot) \times (\mathbb{Z}_0^+, +, \cdot)$, the semiring with pointwise addition and pointwise multiplication. By [4, Lemma 2.1], I is an ideal in $\mathbb{Z}_0^+ \times \mathbb{Z}_0^+$ if and only if $I = I_1 \times I_2$ where I_1, I_2 are ideals in \mathbb{Z}_0^+ .

Theorem 2.5. *Let $I = I_1 \times I_2 \subseteq A = A_1 \times A_2$ be ideals in the semiring $\mathbb{Z}_0^+ \times \mathbb{Z}_0^+$. Then, A is a subtractive extension of I if and only if A_i is a subtractive extension of I_i in \mathbb{Z}_0^+ where $i = 1, 2$.*

Proof. Let A be a subtractive extension of I and $x \in I_1, x+y \in A_1, y \in \mathbb{Z}_0^+$. Then, $(x, 0) \in I$ and $(x, 0) + (y, 0) = (x+y, 0) \in A_1 \times A_2 = A$. Since A is a subtractive extension of I , $(y, 0) \in A$ and hence $y \in A_1$. Now, A_1 is a subtractive extension of I_1 . Similarly, A_2 is a subtractive extension of I_2 . Conversely, suppose that A_i is a subtractive extension of I_i ($i = 1, 2$). Clearly, A is a subtractive extension of I . \square

Lemma 2.6. *Let $I = I_1 \times I_2$ be an ideal of the semiring $\mathbb{Z}_0^+ \times \mathbb{Z}_0^+$ where I_i ($i = 1, 2$) are ideals in the semiring \mathbb{Z}_0^+ . Then, $\bar{I} = \bar{I}_1 \times \bar{I}_2$.*

Theorem 2.7. *Let $I = \langle a_1, a_2, \dots, a_n \rangle \times \langle b_1, b_2, \dots, b_m \rangle$ be an ideal of the semiring $\mathbb{Z}_0^+ \times \mathbb{Z}_0^+$ where $a_i \neq 0, b_j \neq 0$ for some i, j . If $d = (a_1, a_2, \dots, a_n)$ and $d' = (b_1, b_2, \dots, b_m)$, then $\bar{I} = \langle d \rangle \times \langle d' \rangle$.*

Proof. Follows from Lemma 2.1 and Lemma 2.6. \square

3. SUBTRACTIVE EXTENSION OF IDEALS IN SEMIRINGS

In this section, we introduce the concept of closure of an ideal A of a semiring R with respect to an ideal I of R and prove the set of all subtractive extensions of an ideal I of a semiring R is a complete lattice. Also we show that a subtractive extension P of a Q -ideal I in a semiring R is a semiprime ideal if and only if $P/I_{(Q \cap P)}$ is a semiprime ideal in the quotient semiring $R/I_{(Q)}$.

Let $I \subseteq A$ be ideals of a semiring R . Then, we denote $\overline{A}_I = \{x \in R : x + i \in A \text{ for some } i \in I\}$, and will be called the closure of A with respect to I .

We can easily show that (i) $I \subseteq \overline{I} \subseteq \overline{A}_I \subseteq \overline{A}$; (ii) $\overline{A}_A = \overline{A}$ where $I \subseteq A$ are ideals of R .

Example 3.1. Let $I = \{0\} \times \langle 4, 6 \rangle$, $A = \langle 2, 3 \rangle \times \langle 4, 6 \rangle$ be ideals in the semiring $R = \mathbb{Z}_0^+ \times \mathbb{Z}_0^+$. Clearly, $\overline{\{0\}} = \{0\}$. By Lemma 2.1, $\overline{\langle 4, 6 \rangle} = \langle 2 \rangle$ and $\overline{\langle 2, 3 \rangle} = \langle 1 \rangle = \mathbb{Z}_0^+$. Hence, by Lemma 2.6, $\overline{I} = \{0\} \times \langle 2 \rangle$ and $\overline{A} = \mathbb{Z}_0^+ \times \langle 2 \rangle$. An inspection will show that $\overline{A}_I = \langle 2, 3 \rangle \times \langle 2 \rangle$. Now, $I \subsetneq \overline{I} \subsetneq \overline{A}_I \subsetneq \overline{A}$.

Theorem 3.2. Let $I \subseteq A$ be ideals of a semiring R . Then, \overline{A}_I is the smallest subtractive extension of I containing A .

Proof.

- (1) Let $a_1, a_2 \in \overline{A}_I$ and $r \in R$. Then, there exist $i_1, i_2 \in I$ such that $a_1 + i_1, a_2 + i_2 \in A$. Hence, $(a_1 + a_2) + (i_1 + i_2) = a_1 + i_1 + a_2 + i_2 \in A$ where $i_1 + i_2 \in I$. So $a_1 + a_2 \in \overline{A}_I$. Similarly, $ra_1 \in \overline{A}_I$. Hence, \overline{A}_I is an ideal of R .
- (2) Clearly, $A \subseteq \overline{A}_I$.
- (3) Let $i \in I$, $a + i \in \overline{A}_I$, $a \in S$. Then, there exists $i' \in I$ such that $a + i + i' \in A$. Now, $i + i' \in I$ implies $a \in \overline{A}_I$. Hence, \overline{A}_I is a subtractive extension of I .
- (4) Let J be a subtractive extension of I containing A and let $x \in \overline{A}_I$. Then, there exists $i \in I$ such that $x + i \in A \subseteq J$. Since J is a subtractive extension of I , $x \in J$. Hence, $\overline{A}_I \subseteq J$. \square

Corollary 3.3. Let $I \subseteq A$ be ideals of a semiring R . Then,

$$\overline{A}_I = \cap \{J : J \text{ is a subtractive extension of } I \text{ in } R \text{ containing } A\}.$$

Now, we have the following theorem:

Theorem 3.4. Let I, A, B be ideals of a semiring R such that $I \subseteq A, B$. Then,

- (1) A is a subtractive extension of $I \Leftrightarrow \overline{A}_I = A$.
- (2) $\overline{(\overline{A}_I)}_I = \overline{A}_I$.
- (3) $A \subseteq B \Rightarrow \overline{A}_I \subseteq \overline{B}_I$.
- (4) $\overline{(A \cap B)}_I = \overline{A}_I \cap \overline{B}_I$.
- (5) If A, B are subtractive extensions of I , then $A \cap B$ is a subtractive extension of I .
- (6) If J is an ideal of R such that $I \subseteq J \subseteq A$, then $\overline{A}_I \subseteq \overline{A}_J$.

Proof.

- (1) Follows from Theorem 3.2.
- (2) Follows from Theorem 3.2 and (1).
- (3) Let $A \subseteq B$ and let $x \in \overline{A}_I$. Then, there exists $i \in I$ such that $x + i \in A \subseteq B$. Hence, $x \in \overline{B}_I$. Now, $\overline{A}_I \subseteq \overline{B}_I$.
- (4) Since $A \cap B \subseteq A$, $(A \cap B)_I \subseteq \overline{A}_I$. Similarly, $(A \cap B)_I \subseteq \overline{B}_I$. Now, $(A \cap B)_I \subseteq \overline{A}_I \cap \overline{B}_I$. For the other inclusion, let $x \in \overline{A}_I \cap \overline{B}_I$. Then, there exist $i_1, i_2 \in I$ such that $x + i_1 \in A$ and $x + i_2 \in B$. Since $i_2 \in I \subseteq A$ and A is an ideal of R , $x + i_1 + i_2 \in A$. Similarly, $x + i_1 + i_2 \in B$. Now, $x + i_1 + i_2 \in A \cap B$. So $x \in (A \cap B)_I$. Hence, $\overline{A}_I \cap \overline{B}_I \subseteq (A \cap B)_I$.
- (5) By (4), $(A \cap B)_I = \overline{A}_I \cap \overline{B}_I = A \cap B$, since A, B are subtractive extensions of I . So by (1), $A \cap B$ is a subtractive extension of I .
- (6) Suppose that J is an ideal of R such that $I \subseteq J \subseteq A$. Let $x \in \overline{A}_I$. Then, $x + i \in A$ for some $i \in I \subseteq J$. Hence, $x \in \overline{A}_J$. Now, we have $\overline{A}_I \subseteq \overline{A}_J$. \square

Now, by Theorem 3.4, we have

- (i) A is a subtractive ideal of R if and only if $\overline{A} = A$.
- (ii) $\overline{\overline{A}} = \overline{A}$.
- (iii) $A \subseteq B$ implies $\overline{A} \subseteq \overline{B}$.

Corollary 3.5. *Let A, B be ideals of a semiring R . Then, $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$.*

Proof. By using Theorem 3.4 (3) and Theorem 3.4 (6), we have,

$$\begin{aligned} \overline{A \cap B} &= \overline{(A \cap B)_{(A \cap B)}} \\ &\subseteq \overline{A_{(A \cap B)}} \\ &\subseteq \overline{A_A} \\ &= \overline{A}. \end{aligned}$$

Similarly, $\overline{A \cap B} \subseteq \overline{B}$. Now, $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$. \square

The following example shows that the equality in Corollary 3.5 may not hold.

Example 3.6. Let $R = \{0, a, b\}$. Then, R forms a semiring under the following addition and multiplication:

$$\begin{array}{c|ccc} + & 0 & a & b \\ \hline 0 & 0 & a & b \\ a & a & a & b \\ b & b & b & b \end{array} \quad \text{and} \quad \begin{array}{c|ccc} \cdot & 0 & a & b \\ \hline 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \\ b & 0 & 0 & b \end{array}$$

Here $A = \{0, a\}$, $B = \{0, b\}$ are ideals of R . Clearly, $\overline{A \cap B} = \{0\}$. An inspection will show that $\overline{A} = A$ and $\overline{B} = R$. Hence, $\overline{A \cap B} = A$. Now, $\overline{A \cap B} \subsetneq \overline{A} \cap \overline{B}$.

Theorem 3.7. *Let I be an ideal of a semiring R . If $\mathcal{A}(I)$ denotes the set of all subtractive extensions of I in R , then $\mathcal{A}(I)$ is a complete lattice.*

Proof. Clearly, $(\mathcal{A}(I), \subseteq)$ is a partially ordered set. Let $A, B \in \mathcal{A}(I)$. Then, by Theorem 3.4 (5), $A \cap B \in \mathcal{A}(I)$ and by using Theorem 3.2, $\overline{(A + B)}_I \in \mathcal{A}(I)$. Define $A \wedge B = A \cap B$ and $A \vee B = \overline{(A + B)}_I$. Let $C \in \mathcal{A}(I)$ such that $A, B \subseteq C$. Then, $A + B \subseteq C$ and hence $\overline{(A + B)}_I \subseteq \overline{C}_I = C$ since by Theorem 3.4 (1). Now, $\overline{(A + B)}_I$ is a l.u.b. of A, B . Hence, $\mathcal{A}(I)$ is a complete lattice. \square

Corollary 3.8. *Let I be an ideal of a semiring R . If $\mathcal{K}(I)$ denotes the set of all subtractive ideals of R containing I , then $\mathcal{K}(I)$ is a complete lattice.*

Let $I \subseteq A$ be ideals of a semiring R . If I is a Q -ideal of R , then $\tilde{A} = \{x \in R : \text{there exists } q + I \in R/I_{(Q)} \text{ such that } x \in q + I \text{ and } (q + I) \cap A \neq \emptyset\}$, is called the closure of A with respect to $I_{(Q)}$ [4].

In the next lemma we give the relation between \overline{A}_I and \tilde{A} .

Lemma 3.9. *Let I be a Q -ideal of a semiring R and A be an ideal of R with $I \subseteq A$. Then, $\overline{A}_I = \tilde{A}$.*

Proof. Let $x \in \overline{A}_I$. Then, there exists $i_1 \in I$ such that $x + i_1 \in A \dots$ (1). By [2, Lemma 7], there exists $q \in Q$ such that $x \in q + I$. Then, $x = q + i_2$ for some $i_2 \in I$. So $x + i_1 = q + i_2 + i_1 \in q + I$. Thus, $(q + I) \cap A \neq \emptyset$. Hence, $x \in \tilde{A}$. Now, $\overline{A}_I \subseteq \tilde{A}$. For the other inclusion, let $z \in \tilde{A}$. Then, there exists $q + I \in R/I_{(Q)}$ such that $z \in q + I$ and $(q + I) \cap A \neq \emptyset$. So $z = q + i'$ for some $i' \in I$. Let $y = q + i'' \in (q + I) \cap A$ where $i'' \in I$. Since $i' \in I \subseteq A$ and A is an ideal of R , $z + i'' = q + i'' + i' \in A$. Hence, $z \in \overline{A}_I$. Now, $\tilde{A} \subseteq \overline{A}_I$. \square

By Lemma 3.9 and Theorem 3.2, we have the following:

Theorem 3.10 ([4]). *Let I be a Q -ideal of a semiring R and A be an ideal of R with $I \subseteq A$. Then, \tilde{A} is the smallest subtractive extension of I containing A .*

Theorem 3.11. *Let I be a Q -ideal of a semiring R and P a subtractive extension of I . Then, P is a semiprime ideal of R if and only if $P/I_{(Q \cap P)}$ is a semiprime ideal of $R/I_{(Q)}$.*

Proof. Let P be a semiprime ideal of R . Suppose that $q + I \in R/I_{(Q)}$ and $(q + I) \odot (q + I) = q' + I \in P/I_{(Q \cap P)}$ where $q' \in Q \cap P$ is a unique element such

that $q^2 + I \subseteq q' + I$. So $q^2 = q' + i$ for some $i \in I$. Now, $q^2 \in P$ implies $q \in P$. Hence, $q + I \in P/I_{(Q \cap P)}$. Conversely, suppose that $P/I_{(Q \cap P)}$ is a semiprime ideal of $R/I_{(Q)}$. Let $a^2 \in P$ where $a \in R$. Since I is a Q -ideal of R , there exist unique $q, q' \in Q$ such that $a \in q + I$ and $a^2 \in (q + I) \odot (q + I) = q' + I$, where $q^2 + I \subseteq q' + I$. Thus, $a^2 = q' + i'$ for some $i' \in I$. Since P is a subtractive extension of I , $q' \in P$. Hence, $(q + I) \odot (q + I) = q' + I \in P/I_{(Q \cap P)}$. Since $P/I_{(Q \cap P)}$ is a semiprime ideal, $q + I \in P/I_{(Q \cap P)}$. Now, $a \in q + I \Rightarrow a = q + i''$ for some $i'' \in I \Rightarrow a \in P$ as $q \in Q \cap P \subseteq P$. \square

REFERENCES

- [1] S. E. Atani, *The ideal theory in quotients of commutative semirings*, Glas. Mat. Ser. III, 42 (62) (2007), no. 2, 301–308.
- [2] P. J. Allen, *A fundamental theorem of homomorphism for semirings*, Proc. Amer. Math. Soc., 21 (1969), 412–416.
- [3] P. J. Allen and L. Dale, *Ideal theory in the semiring \mathbb{Z}_0^+* , Publ. Math. Debrecen, 22 (3-4) (1975), 219–224.
- [4] J. N. Chaudhari and D. R. Bonde, *Ideal theory in quotient semirings*, Thai J. Math., 12 (1) (2014), 95–101.
- [5] J. N. Chaudhari and V. Gupta, *Weak primary decomposition theorem for right Noetherian semirings*, Indian J. Pure Appl. Math., 25 (6) (1994), 647–654.
- [6] L. Dale, *The k -closure of monic and monic free ideals in a polynomial semiring*, Proc. Amer. Math. Soc., 64 (2) (1977), 219–226.
- [7] J. S. Golan, *Semiring and Their Applications*, Kluwer Academic publisher Dordrecht, 1999.
- [8] V. Gupta and J. N. Chaudhari, *Some remarks on semirings*, Rad. Mat., 12 (2003), 13–18.
- [9] M. K. Sen and M. R. Adhikari, *On k -ideals of semirings*, Internat. J. Math. Math. Sci., 15 (2) (1992), 347–350.

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