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SLIGHTLY GENERALIZED β-CONTINUOUS FUNCTIONS

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Abstract. A new class of functions, called slightly generalized β-continuous functions is introduced. Basic properties of slightly generalized β-continuous functions are studied. The class of slightly generalized β-continuous functions properly includes the class of slightly β-continuous functions and generalized β-continuous functions. Also, by using slightly generalized β-continuous functions, some properties of domain/range of functions are characterized.

1. Introduction and preliminaries

Slightly β-continuous functions were introduced by Noiri [9] in 2000 and next have been developed by Tahiliani [13]. Dontchev [4] introduced the notion of generalized β-continuous functions and investigated some of their basic properties and further Tahiliani [12] introduced the notion of β-generalized β-continuous functions. In this paper, we defined slightly generalized β-continuous functions and show that the class of slightly generalized β-continuous functions properly includes the class of slightly β-continuous functions and generalized β-continuous functions. Second we obtain some new results on $g\beta$-closed sets and investigate basic properties of slightly generalized β-continuous functions concerning composition and restriction.

Finally, we study the behaviour of some separation axioms, related properties and $G\beta O$-compactness, $G\beta O$-connectedness under slightly generalized β-continuous functions. Relationship between generalized β-continuous functions and $G\beta O$-connected spaces are investigated. In particularly, it is shown that slightly generalized β-continuous image of $G\beta O$-connected spaces is connected.

Throughout this paper, (X, τ) and (Y, σ) (or X and Y) represents a non empty topological space on which no separation axioms are assumed, unless

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otherwise mentioned.The closure and interior of $A \subseteq X$ will be denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ respectively.

Definition 1.1.

(i) *A subset A of a space X is called β-open* [1] *if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$. The complement of β-open set is β-closed* [1]*. The intersection of all β-closed sets containing A is called β-closure of A and is denoted by $\beta\,\mathrm{Cl}(A)$. Also A is said to be β-clopen* [9] *if it is β-open and β-closed. The largest β-open set contained in A (denoted by $\beta\,\mathrm{Int}(A)$) is called β-interior* [2] *of A.*

(ii) *A subset A of a space X is said to be generalized closed* [6] *(briefly g-closed) if $\mathrm{Cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.*

(iii) *A subset A of a space X is said to be generalized semi preclosed* [4] *(briefly gsp-closed) or $g\beta$-closed* [4] *if $\beta\,\mathrm{Cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.*

(iv) *Generalized semi-preopen* [4] *(briefly $g\beta$-open) if $F \subseteq \beta\,\mathrm{Int}(A)$ whenever $F \subseteq A$ and F is closed in X. Also it is a complement of $g\beta$-closed set. If A is both $g\beta$-closed and $g\beta$-open, then it is said to be $g\beta$-clopen.*

In this note, the family of all open (resp. g-open, g β-open, clopen) sets of a space X is denoted by $O(X)$ (resp. $GO(X)$, $G\beta O(X)$, $CO(X)$) and the family of $g\beta$-open(resp. clopen) sets of X containing x is denoted by $G\beta O(X, x)$ (resp. $CO(X, x)$).

Definition 1.2. *A function $f : X \to Y$ is called:*

(i) *gsp-continuous* [4] *or $g\beta$-continuous (resp. gsp-irresolute* [4] *or $g\beta$-irresolute) if $f^{-1}(F)$ is $g\beta$-closed in X for every closed (resp. $g\beta$-closed) set F of Y.*

(ii) *Slightly continuous* [10] *(resp. slightly β-continuous* [9]*) if for each $x \in X$ and each clopen set V of Y containing $f(x)$, there exists a open(resp. β-open) set U such that $f(U) \subseteq V$.*

(iii) *gsp-irresolute* [4] *or $g\beta$-irresolute* [12] *if $f^{-1}(F)$ is $g\beta$-closed in X for every $g\beta$-closed set F of Y.*

(iv) *Pre-β-closed* [7] *if the image of each β-closed set in X is β-closed in Y.*

(v) *$g\beta$-homeomorphism if it is bijective, $g\beta$-irresolute and its inverse f^{-1} is $g\beta$-irresolute.*

2. Slightly generalized β-continuous functions

Definition 2.1. *A function $f : X \to Y$ is called slightly generalized β-continuous (briefly sl.$g\beta$-continuous) if the inverse image of every clopen set in Y is $g\beta$-open in X.*

The proof of the following theorem is straightforward and hence omitted.

Theorem 2.1. *For a function $f : X \to Y$, the following statements are equivalent*:

(i) *f is slightly $g\beta$-continuous.*

(ii) *Inverse image of every clopen subset of Y is $g\beta$-open in X.*

(iii) *Inverse image of every clopen subset of Y is $g\beta$-clopen in X.*

Obviously, slight β-continuity implies $sl.g\beta$-continuity and $g\beta$-continuity implies $sl.g\beta$-continuity. The following example shows that the implications are not reversible.

Example 2.1. Let $X = \{a, b, c\}$, $\tau = \{\varnothing, X, \{a\}\}$ and $Y = \{p, q\}$, $\sigma = \{\varnothing, Y, \{p\}, \{q\}\}$ be the topologies on X and Y respectively. Let $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = f(c) = q$ and $f(b) = p$. Then f is slightly $g\beta$-continuous but not slightly β-continuous.

Example 2.2. Let $X = \{a, b, c\}$ and let $\tau = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\varnothing, X, \{c\}\}$ be the topologies on X respectively. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f is slightly $g\beta$-continuous but not $g\beta$-continuous.

A space is called locally discrete if every open subset is closed [3]. Also, a space is called as semi-pre-$T_{1/2}$ [4] if every $g\beta$-closed subset of it is β closed.

The next two theorems are immediate of the definitions of a locally discrete and semi-pre-$T_{1/2}$ space.

Theorem 2.2. *If $f : X \to Y$ is slightly $g\beta$-continuous and Y is locally discrete, then f is $g\beta$-continuous.*

Theorem 2.3. *If $f : X \to Y$ is slightly $g\beta$-continuous and X is semi-pre-$T_{1/2}$ space, then f is slightly β-continuous.*

3. Basic properties of slightly generalized β-continuous functions

Definition 3.1. *The intersection of all $g\beta$-closed sets containing a set A is called $g\beta$-closure of A and is denoted by $g\beta\,\mathrm{Cl}(A)$.*

Remark 3.1. It is obvious that $g\beta\,\mathrm{Cl}(A)$ is $g\beta$-closed and A is $g\beta$-closed if and only if $g\beta\,\mathrm{Cl}(A) = A$.

Lemma 3.1. *Let A be a $g\beta$-open set and B be any set in X. If $A \cap B = \varnothing$, then $A \cap g\beta\,\mathrm{Cl}(B) = \varnothing$.*

Proof. Suppose that $A \cap g\beta\,\mathrm{Cl}(B) \neq \varnothing$ and $x \in A \cap g\beta\,\mathrm{Cl}(B)$. Then $x \in A$ and $x \in g\beta\,\mathrm{Cl}(B)$ and from the definition of $g\beta\,\mathrm{Cl}(B), A \cap B \neq \varnothing$. (Same as Theorem 2.3 [2] by replacing β-open set by gβ-open). This is contrary to hypothesis.

For a subset A of space X, the kernel of A [8], denoted by $\ker(A)$, is the intersection of all open supersets of A. □

Proposition 3.1. *A subset A of X is $g\beta$-closed if and only if $\beta\,\mathrm{Cl}(A) \subseteq \ker(A)$.*

Proof. Since A is $g\beta$-closed, $\beta\,\mathrm{Cl}(A) \subseteq U$ for any open set U with $A \subseteq U$ and hence $\beta\,\mathrm{Cl}(A) \subseteq \ker(A)$. Conversely, let U be any open set such that $A \subseteq U$. By hypothesis, $\beta\,\mathrm{Cl}(A) \subseteq \ker(A) \subseteq U$ and hence A is $g\beta$-closed. □

Dontchev [4] has proved that the intersection of two $g\beta$-closed sets is generally not a $g\beta$-closed set and the union of two $g\beta$-open sets is generally not a $g\beta$-open set.

Proposition 3.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If f is slightly $g\beta$-continuous, then for each point $x \in X$ and each clopen set V containing $f(x)$, there exists a $g\beta$-open set U containing x such that $f(U) \subseteq V$.*

Proof. Let $x \in X$ and V be a clopen set such that $f(x) \in V$. Since f is slightly $g\beta$-continuous, $f^{-1}(V)$ is $g\beta$-open set in X. If we put $U = f^{-1}(V)$, we have $x \in U$ and $f(U) \subseteq V$. □

Let (X, τ) be a topological space. The quasi-topology on X is the topology having as base all clopen subsets of (X, τ). The open (resp. closed) subsets of the quasi-topology are said to be quasi-open (resp. quasi-closed). A point x of a space X is said to be quasi closure of a subset A of X, denoted by $\mathrm{Cl}_q\, A$, if $A \cap U \neq \varnothing$ for every clopen set U containing x. A subset A is said to be quasi closed if and only if $A = \mathrm{Cl}_q\, A$ [11]. If the closure of A in topological space coincides with $g\beta\,\mathrm{Cl}(A)$, then it is denoted by (X, c).

Proposition 3.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent*:

- (i) *For each point $x \in X$ and each clopen set V containing $f(x)$, there exists a $g\beta$-open set U containing x such that $f(U) \subseteq V$.*
- (ii) *For every subset A of $X, f(g\beta\,\mathrm{Cl}(A)) \subseteq \mathrm{Cl}_q(f(A))$.*
- (iii) *The map $f : (X, c) \to (Y, \sigma)$ is slightly-continuous.*

Proof. **(i)⇒(ii).** Let $y \in f(g\beta\,\mathrm{Cl}(A))$ and V be any clopen nbd of y. Then there exists a point $x \in X$ and a $g\beta$-open set U containing x such that $f(x) = y$, $x \in g\beta\,\mathrm{Cl}(A)$ and $f(U) \subseteq V$. Since $x \in g\beta\,\mathrm{Cl}(A)$, $U \cap A \neq \varnothing$ holds and hence $V \cap f(A) \neq \varnothing$. Therefore we have $y = f(x) \in \mathrm{Cl}_q(f(A))$.

(ii)⇒(i). Let $x \in X$ and let V be a clopen set with $f(x) \in V$. Let $A = f^{-1}(Y \setminus V)$, then $x \notin A$. Since $f(g\beta\operatorname{Cl}(A)) \subseteq \operatorname{Cl}_q(f(A)) \subseteq \operatorname{Cl}_q(Y \setminus V) = Y \setminus V$, it is shown that $g\beta\operatorname{Cl}(A) = A$. Then since $x \notin g\beta\operatorname{Cl}(A)$, there exists $g\beta$-open set U containing x such that $U \cap A = \varnothing$ and hence $f(U) \subseteq f(X \setminus A) \subseteq V$.

(ii)⇒(iii). Suppose that (ii) holds and let V be any clopen subset of Y. Since $f(g\beta\operatorname{Cl}(f^{-1}(V))) \subseteq \operatorname{Cl}_q(f(f^{-1}(V))) \subseteq \operatorname{Cl}_q(V) = V$, it is shown that $\beta\operatorname{Cl}(f^{-1}(V)) = f^{-1}(V)$ and hence we have $f^{-1}(V)$ is gβ-closed in (X, τ) and hence $f^{-1}(V)$ is closed in (X, c).

(iii)⇒(ii). Conversely, let $y \in f(g\beta\operatorname{Cl}(A))$ and V be any clopen nbd of y. Then there exists a point $x \in X$ such that $f(x) = y$ and $x \in g\beta\operatorname{Cl}(A)$. Since f is slightly continuous, $f^{-1}(V)$ is open in (X, c) and so $g\beta$-open set containing x. Since $x \in g\beta\operatorname{Cl}(A)$, $f^{-1}(V) \cap A \neq \varnothing$ holds and hence $V \cap f(A) \neq \varnothing$. Therefore, we have $y = f(x) \in \operatorname{Cl}_q(f(A))$. □

Now we investigate some basic properties of slightly $g\beta$-continuous functions concerning composition and restriction. The proofs of first three results are straightforward and hence omitted.

Theorem 3.1. *If $f : X \to Y$ is $g\beta$-irresolute and $g : Y \to Z$ is slightly $g\beta$-continuous, then $g \circ f : X \to Z$ is slightly $g\beta$-continuous.*

Theorem 3.2. *If $f : X \to Y$ is slightly $g\beta$-continuous and $g : Y \to Z$ is continuous, then $g \circ f : X \to Z$ is slightly $g\beta$-continuous.*

Corollary 3.1. *Let $\{X_i : i \in I\}$ be any family of topological spaces. If $f : X \to \prod X_i$ is sl.$g\beta$-continuous mapping, then $P_i \circ f : X \to X_i$ is sl.$g\beta$ continuous for each $i \in I$, where P_i is the projection of ΠX_i onto X_i.*

Lemma 3.2. *Let $f : X \to Y$ be bijective, continuous and pre-β-closed. Then for every $g\beta$-open set A of X, $f(A)$ is $g\beta$-open in Y.*

Theorem 3.3. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is bijective, continuous and pre-β-closed and if $g \circ f : X \to Z$ is sl.$g\beta$ continuous, then g is sl.$g\beta$-continuous.*

Proof. Let V be a clopen subset of Z. Then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g\beta$-open in X. Then by above Lemma, $g^{-1}(V) = f(f^{-1}(g^{-1}(V)))$ is $g\beta$-open in Y. □

Combining Theorem 3.1 and 3.3, we obtain the following result.

Corollary 3.2. *Let $f : X \to Y$ be a bijective $g\beta$-homeomorphism and let $g : Y \to Z$ be a function.Then $gof : X \to Z$ is sl.$g\beta$-continuous if and only if g is sl.$g\beta$-continuous.*

We know that for a $g\beta$-closed set A and open set F, the intersection $A \cap F$ is $g\beta$-closed set relative to F ([4, Theorem 3.17(ii)]).Thus we have the following result.

Theorem 3.4. *If $f : X \to Y$ is slightly.$g\beta$-continuous and A is open subset of X, then $f|_A : A \to Y$ is slightly $g\beta$-continuous.*

Proof. Let V be a clopen subset of Y. Then $(f|_A)^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ is $g\beta$-closed and A is open, $(f|_A)^{-1}(V)$ is $g\beta$-closed in the relative topology of A.

4. Some application theorems

Definition 4.1. *A space is called*

(i) *$g\beta$-T_2 (resp. ultra Hausdorff or UT2* [10]*) if every two distinct points of X can be separated by disjoint $g\beta$-open(resp. clopen) sets.*

(ii) *$G\beta O$-compact* [12] *(resp. mildly compact* [11]*) if every $g\beta$-open (resp. clopen) cover has a finite subcover.*

Let $X = \{a, b, c\}$ and $\tau = \{\varnothing, X, \{a\}\}$ be the topology on X. Then (X, τ) is $g\beta$-T_2 but, if we take $X = \{a, b, c\}$ and $\tau = \{\varnothing, X, \{a\}, \{a, b\}\}$, then (X, τ) is not $g\beta$-T_2.

The following theorem gives a characterization of $g\beta$-T_2 spaces and is an analogous to that in general topology, hence its proof is omitted.

Theorem 4.1. *A space X is $g\beta$-T_2 if and only if for every point x in X, $\{x\} = \cap\{F : F$ is $g\beta$-closed nbd of $x\}$.*

Theorem 4.2. *If $f : X \to Y$ is sl.$g\beta$-continuous injection and Y is UT_2, then X is $g\beta$-T_2.*

Proof. Let $x_1, x_2 \in X$ and $x_1 \neq x_2$. Then since f is injective and Y is UT_2, $f(x_1) \neq f(x_2)$ and there exist $V_1, V_2 \in CO(Y)$ such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $V_1 \cap V_2 = \varnothing$. Since f is $sl.g\beta$-continuous, $x_i \in f^{-1}(V_i) \in G\beta O(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \varnothing$. Thus X is $g\beta$-T_2. □

Theorem 4.3. *If $f : X \to Y$ is sl.$g\beta$-continuous surjection, and X is $G\beta O$-compact, then Y is mildly compact.*

Proof. Let $\{V_a : V_\alpha \in CO(Y), \alpha \in I\}$ be a cover of Y. Since f is $sl.g\beta$-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ be $g\beta$-cover of X so there is a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ since f is surjective. Thus Y is mildly compact. □

Theorem 4.4. *If $f : X \to Y$ is a sl.$g\beta$-continuous injection and Y is UT_2, then the graph $G(f)$ of f is $g\beta$-closed in the product space $X \times Y$.*

Proof. Let $(x, y) \notin G(f)$, then $y \neq f(x)$. Since Y is UT_2, there exist $V_1, V_2 \in CO(Y)$ such that $y \in V_1$ and $f(x) \in V_2$ such that $V_1 \cap V_2 = \varnothing$. Since f is slightly. $g\beta$-continuous, by Proposition 3.2, there exists $U \in G\beta O(X, x)$ such that $f(U) \subseteq V_2$. Therefore, $f(U) \cap V_1 = \varnothing$ and hence $(U \times V_1) \cap G(f) = \varnothing$. Since $U \in G\beta O(X, x)$ and $V_1 \in CO(Y, y)$, $(x, y) \in (U \times V_1) \in G\beta O(X \times Y)$ ([12, Lemma 4.3]). Thus we obtain $(x, y) \notin g\beta \operatorname{Cl}(G(f))$ (Remark 3.1).

Theorem 4.5. *If $f : X \to Y$ is a sl.$g\beta$-continuous injection and Y is UT_2, then $A = \{(x_1, x_2) : f(x_1) = f(x_2)\}$ is $g\beta$-closed in the product space $X \times X$.*

Proof. Let $(x_1, x_2) \notin A$, then $f(x_1) \neq f(x_2)$. Since Y is UT_2, there exist $V_1, V_2 \in CO(Y)$ such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $V_1 \cap V_2 = \varnothing$. Since f is sl.$g\beta$ continuous, $x_i \in f^{-1}(V_i) \in G\beta O(X)$ for $i = 1, 2$. Therefore, $(f^{-1}(V_1) \times f^{-1}(V_2)) \cap A = \varnothing$. Since $(x_1, x_2) \in (f^{-1}(V_1) \times f^{-1}(V_2)) \in G\beta O(X \times X)$ ([12, Lemma 4.3]). We obtain $(x_1, x_2) \notin g\beta \operatorname{Cl}(A)$ (Remark 3.1). □

We shall continue to work by generalizing the well known theorems in general topology.

Recall that a space X is submaximal if every dense set is open and it is said to be extremally disconnected if the closure of every open set is open.

Lemma 4.1. *If X is submaximal and extremally disconnected, then every β-open set in X is open* [5].

Remark 4.1. By Lemma 4.1, we can say that every $g\beta$-open set in X is g-open as every β-open set is $g\beta$-open and every open set is g-open.

Theorem 4.6. *If $f, g : X \to Y$ is a sl.$g\beta$-continuous, Y is UT_2, X is submaximal and extremally disconnected, then $A = \{x \in X : f(x) = g(x)\}$ is $g\beta$-closed.*

Proof. Let $x \notin A$, then $f(x) \neq g(x)$. Since Y is UT_2, there exist $V_1, V_2 \in CO(Y)$ such that $f(x) \in V_1$ and $g(x) \in V_2$ and $V_1 \cap V_2 = \varnothing$. Since f and g are sl.$g\beta$-continuous, $f^{-1}(V_1)$ and $g^{-1}(V_2)$ are $g\beta$-open and hence g-open since X is submaximal and extremally disconnected (Remark 4.1) with $x \in f^{-1}(V_1) \cap g^{-1}(V_2)$.

Let $U = f^{-1}(V_1) \cap g^{-1}(V_2)$. Then U is a g-open set ([6, Theorem 2.4]) and $U \cap A = \varnothing$ and so $x \notin g\beta \operatorname{Cl}(A)$. □

Definition 4.2. *A subset of a space X is said to be $g\beta$-dense if its $g\beta$-closure equals X.*

The next corollary is a generalization of the well known principle of extension of the identity.

Corollary 4.1. *Let f, g be sl-$g\beta$-continuous from a space X into a UT_2-space Y. If f and g agree on $g\beta$-dense set of X, then $f = g$ everywhere.*

Definition 4.3. *Let A be a subset of $X.A$ mapping $r : X \to A$ is called sl.$g\beta$-continuous retraction if X is sl.$g\beta$-continuous and the restriction $r|_A$ is the identity mapping on A.*

Theorem 4.7. *Let A be a subset of X and $r : X \to A$ be a sl.$g\beta$-continuous retraction. If X is UT_2, then A is $g\beta$-closed set of X.*

Proof. Suppose that A is not $g\beta$-closed. Then there exists a point x in X such that $x \in g\beta\,\mathrm{Cl}(A)$ but $x \notin A$. It follows that $r(x) \neq x$ because r is $sl.g\beta$-continuous retraction. Since X is UT_2, there exist disjoint clopen sets U and V such that $x \in U$ and $r(x) \in V$. Since $r(x) \in A$, $r(x) \in V \cap A$ and $V \cap A$ is clopen set in A. Now let W be arbitrary $g\beta$-nbhd of x. Then $W \cap U$ is a $g\beta$-nbhd of x. Since $x \in g\beta\,\mathrm{Cl}(A), (W \cap U) \cap A \neq \varnothing$. Therefore, there exists a point y in $W \cap U \cap A$. Since $y \in A$, we have $r(y) = y \in U$ and hence $r(y) \notin V \cap A$. This implies $r(W) \not\subset V \cap A$ because $y \in W$. This is contrary to $sl.g\beta$-continuity of r from Proposition 3.2. Hence A is $g\beta$-closed. ☐

Definition 4.4. *A space X is called $G\beta O$-connected provided X is not the union of two disjoint, non-empty $g\beta$-open sets.*

Theorem 4.8. *If $f : X \to Y$ is sl.$g\beta$-continuous surjection, and X is $G\beta O$-connected, then Y is connected.*

Proof. Assume that Y is disconnected. Then there exist disjoint, non-empty clopen sets U and V for which $Y = U \cup V$. Therefore, $X = f^{-1}(U) \cup f^{-1}(V)$ is the union of two disjoint, $g\beta$-open nonempty sets and hence is not $G\beta O$-connected.

Slightly $g\beta$-continuity turns out to be a very natural tool for relating $G\beta O$-connected spaces to connected spaces. Much of the theory developed by Tahiliani [13] on β-connected sets and slightly β-continuous functions can be modified and extended to $G\beta O$-connected sets and slightly generalized β-continuous functions. In Theorem 4.8, we have seen that the $sl.g\beta$-continuous image of a $G\beta O$-connected space is connected but that a $sl.g\beta$-continuous function is not necessarily a $G\beta O$-connected function which is defined below.

Definition 4.5. *A function $f : X \to Y$ is called $G\beta O$-connected if the image of every $G\beta O$-connected subset of X is a connected subset of Y.*

The following example shows that a $sl.g\beta$-continuous function is not necessarily $G\beta O$-connected.

Example 4.1. Let X be a set containing three distinct elements p, q, r. For each $x \in X$, let $\sigma_x = \{U \subseteq X : U = \varnothing \text{ or } x \in U\}$ be the corresponding particular point topology. Let $f : (X, \sigma_p) \to (X, \sigma_q)$ be the identity map. Since (X, σ_q) is connected, f is slightly $g\beta$-continuous. The set $\{p, r\}$ is $G\beta O$-connected in (X, σ_p) as the $g\beta$-open sets of (X, σ_x) are precisely the open sets. However $f(\{p, r\}) = \{p, r\}$ is not connected in (X, σ_q). It follows that f is not $G\beta O$-connected.

Next we show by the example that a $G\beta O$-connected function need not be sl.$g\beta$-continuous.

Example 4.2. Let $X = \{1/n : n \in N\} \cup \{0\}$ and let σ be the usual relative topology on X. Let $Y = \{0, 1\}$ and let τ be the discrete topology on Y. Define $f : (X, \sigma) \to (Y, \tau)$ as $f(1/n) = 0$ for every $n \in N$ and $f(0) = 1$. It can be seen that the $g\beta$-open sets in (X, σ) are the precisely the open sets. Then follows that f is $G\beta O$-connected but not slightly $g\beta$-continuous.

Thus we established that slight.$g\beta$-continuity and $G\beta O$-connectedness are independent.

Definition 4.6. *A space X is said to be $G\beta O$-connected between the subsets A and B of X provided there is no $g\beta$-clopen set F for which $A \subseteq F$ and $F \cap B = \varnothing$.*

Definition 4.7. *A function $f : X \to Y$ is said to be set $G\beta O$-connected if whenever X is $G\beta O$-connected between subsets A and B of X, then $f(X)$ is connected between $f(A)$ and $f(B)$ with respect to the relative topology on $f(X)$.*

Theorem 4.9. *A function $f : X \to Y$ is set $G\beta O$-connected if and only if $f^{-1}(F)$ is $g\beta$-clopen in X for every clopen set F of $f(X)$ (with respect to the relative topology on $f(X)$).*

Proof. The proof is obtained by following similar arguments as in ([13, Theorem 3.4]).

Obviously, every $sl.g\beta$-continuous surjective function is set $G\beta O$-connected. On the other hand, it can be easily shown that every set $G\beta O$-connected function is $sl.g\beta$-continuous. Thus we have seen that in the class of surjective functions, $sl.g\beta$-continuity and set $G\beta O$-connectedness coincide. The following example shows that in general sl.gβ-continuity is not equivalent to set $G\beta O$-connectedness.

Example 4.3. Let $X = \{0, 1\}$ and $\tau = \{\varnothing, X, \{1\}\}$. Let $Y = \{a, b, c\}$ and $\sigma = \{\varnothing, Y, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined as $f(0) = a$ and $f(1) = b$. Then f is slightly $g\beta$-continuous by Definition 2.1 but not set $G\beta O$-connected as $\{a\}$ is clopen in the relative topology on $f(X)$ but $f^{-1}\{a\} = \{0\}$ which is not $g\beta$-open in (X, τ).

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