SARAJEVO JOURNAL OF MATHEMATICS
Vol.8 (21) (2012), 235–244
DOI: 10.5644/SJM.08.2.05

COEFFICIENT ESTIMATES FOR SAKAGUCHI TYPE FUNCTIONS

HALIT ORHAN, NIHAT YAGMUR AND MURAT ÇAĞLAR

Abstract. Let $S^n_{\lambda,\mu}(\alpha,t)$ be the class of normalized analytic functions defined in the open unit disk satisfying

$$\Re\left(\frac{(1-t)z\left(D^n_{\lambda,\mu}f(z)\right)'}{D^n_{\lambda,\mu}f(z)-D^n_{\lambda,\mu}f(tz)}\right)>\alpha,\quad |t|\leq 1,\ t\neq 1$$

for some $\alpha(0\leq\alpha<1)$ and $D^n_{\lambda,\mu}$ is a linear multiplier differential operator defined by the authors in [2]. The object of the present paper is to discuss some properties of functions $f(z)$ belonging to the classes $S^n_{\lambda,\mu}(\alpha,t)$ and $T^n_{\lambda,\mu}(\alpha,t)$ where $f(z)\in T^n_{\lambda,\mu}(\alpha,t)$ if and only if $zf'(z)\in S^n_{\lambda,\mu}(\alpha,t)$.

1. Introduction

Let $\mathcal{A}$ denote the family of functions f of the form

$$f(z)=z+\sum_{k=2}^{\infty}a_kz^k \tag{1.1}$$

which are analytic in the open unit disk $\mathcal{U}=\{z:|z|<1\}$. For $f(z)$ belongs to $\mathcal{A}$, the *multiplier differential operator* $D^n_{\lambda,\mu}f$ was defined by the authors in [2] as follows

$$\begin{aligned}
D^0_{\lambda,\mu}f(z)&=f(z)\\
D^1_{\lambda,\mu}f(z)&=D_{\lambda,\mu}f(z)=\lambda\mu z^2(f(z))''+(\lambda-\mu)z(f(z))'+(1-\lambda+\mu)f(z)\\
D^2_{\lambda,\mu}f(z)&=D_{\lambda,\mu}\left(D^1_{\lambda,\mu}f(z)\right)\\
&\vdots\\
D^n_{\lambda,\mu}f(z)&=D_{\lambda,\mu}\left(D^{n-1}_{\lambda,\mu}f(z)\right)
\end{aligned}$$

2000 *Mathematics Subject Classification.* 30C45.

Key words and phrases. Analytic function, coefficient estimates, Sakaguchi function, linear multiplier differential operator.

where $\lambda \geqslant \mu \geqslant 0$ and $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

If f is given by (1.1) then from the definition of the operator $D^n_{\lambda,\mu} f(z)$ it is easy to see that

$$D^n_{\lambda,\mu} f(z) = z + \sum_{k=2}^{\infty} [1 + (\lambda\mu k + \lambda - \mu)(k-1)]^n a_k z^k. \quad (1.2)$$

It should be remarked that the $D^n_{\lambda,\mu}$ is a generalization of many other linear operators considered earlier by different authors. In particular, for $f \in \mathcal{A}$ we have the following:

- $D^n_{1,0} f(z) \equiv D^n f(z)$ the operator investigated by Sălăgean (see [4]).
- $D^n_{\lambda,0} f(z) \equiv D^n_\lambda f(z)$ the operator studied by Al-Oboudi (see [3]).
- $D^n_{\lambda,\mu} f(z)$ the operator firstly considered for $0 \leqslant \mu \leqslant \lambda \leqslant 1$, by Răducanu and Orhan (see [1]).

A function $f(z) \in \mathcal{A}$ is said to be in the class $S^n_{\lambda,\mu}(\alpha, t)$ if it satisfies

$$\Re\left(\frac{(1-t)z\left(D^n_{\lambda,\mu} f(z)\right)'}{D^n_{\lambda,\mu} f(z) - D^n_{\lambda,\mu} f(tz)}\right) > \alpha, \quad |t| \leq 1,\ t \neq 1 \quad (1.3)$$

for all $z \in \mathcal{U}$ and some $\alpha (0 \leq \alpha < 1)$.

We also denote by $T^n_{\lambda,\mu}(\alpha, t)$ the subclass of $\mathcal{A}$ consisting of all functions $f(z)$ such that $zf'(z) \in S^n_{\lambda,\mu}(\alpha, t)$. The class $S^0_{\lambda,\mu}(0, -1)$ was introduced by Sakaguchi [5]. Therefore, a function $f(z) \in S^0_{\lambda,\mu}(\alpha, -1)$ is called Sakaguchi function of order α (see [6] and [8]). Further, the class $S^0_{\lambda,\mu}(\alpha, t)$ was introduced and studied by Owa et al. [7]. Various Sakaguchi type functions were investigated and studied by many authors including ([9], [10], [11]). We note that $S^0_{\lambda,\mu}(0, -1)$ is the class of starlike functions with respect to symmetric points in $\mathcal{U}$. Also $S^0_{\lambda,\mu}(\alpha, 0) = S^*(\alpha)$ and $T^0_{\lambda,\mu}(\alpha, 0) = C(\alpha)$ which are, respectively, the familiar classes of starlike functions of order $\alpha (0 \leq \alpha < 1)$ and convex functions of order $\alpha (0 \leq \alpha < 1)$. Incidentally the class of uniformly starlike functions introduced by Goodman [12] as follows

$$UST = \left\{ f(z) \in \mathcal{A} : \Re\left(\frac{(z-\zeta)f'(z)}{f(z) - f(\zeta z)}\right) > 0 \right\}, \quad (z, \zeta) \in \mathcal{U} \text{x} \mathcal{U}.$$

Ronning [13] showed the following important result.

Remark 1.1. $f(z) \in UST$ if and only if for every $z \in \mathcal{U}$, $|t| = 1$

$$\Re\left(\frac{(1-t)zf'(z)}{f(z) - f(tz)}\right) > 0.$$

Now we will give some results for functions belonging to the classes $S^m_{0,\lambda,\mu}(\alpha, t)$ and $T^m_{0,\lambda,\mu}(\alpha, t)$.

2. $S^n_{0,\lambda,\mu}(\alpha,t)$ and $T^n_{0,\lambda,\mu}(\alpha,t)$

Theorem 2.1. *If $f(z)\in\mathcal{A}$ satisfies*

$$\sum_{k=2}^{\infty} A_k^n \left\{|k-u_k|+(1-\alpha)\left|u_k\right|\right\}|a_k| \le 1-\alpha,$$

$$u_k = 1+t+t^2+\cdots+t^{k-1},\ t(|t|\le 1, t\neq 1) \quad (2.1)$$

for some $\alpha(0\le\alpha<1)$ then $f(z)\in S^n_{\lambda,\mu}(\alpha,t)$, where

$$A_k^n = [1+(\lambda\mu k+\lambda-\mu)(k-1)]^n.$$

Proof. To prove Theorem 2.1, we show that if $f(z)$ satisfies (2.1) then

$$\left|\frac{(1-t)z\left(D^n_{\lambda,\mu}f(z)\right)'}{D^n_{\lambda,\mu}f(z)-D^n_{\lambda,\mu}f(tz)}-1\right|<1-\alpha.$$

Evidently, since

$$\frac{(1-t)z\left(D^n_{\lambda,\mu}f(z)\right)'}{D^n_{\lambda,\mu}f(z)-D^n_{\lambda,\mu}f(tz)}-1=\frac{z+\sum\limits_{k=2}^{\infty}kA_k^n a_k z^k}{z+\sum\limits_{k=2}^{\infty}kA_k^n u_k a_k z^k}-1=\frac{\sum\limits_{k=2}^{\infty}(k-u_k)A_k^n a_k z^{k-1}}{1+\sum\limits_{k=2}^{\infty}A_k^n u_k a_k z^{k-1}}$$

we see that

$$\left|\frac{(1-t)z\left(D^n_{\lambda,\mu}f(z)\right)'}{D^n_{\lambda,\mu}f(z)-D^n_{\lambda,\mu}f(tz)}-1\right|\le\frac{\sum\limits_{k=2}^{\infty}A_k^n\left|k-u_k\right||a_k|}{1-\sum\limits_{k=2}^{\infty}A_k^n\left|u_k\right||a_k|}.$$

Therefore, if $f(z)$ satisfies (2.1), then we have

$$\left|\frac{(1-t)z\left(D^n_{\lambda,\mu}f(z)\right)'}{D^n_{\lambda,\mu}f(z)-D^n_{\lambda,\mu}f(tz)}-1\right|<1-\alpha.$$

This completes the proof of Theorem 2.1. □

Theorem 2.2. *If $f(z)\in\mathcal{A}$ satisfies*

$$\sum_{k=2}^{\infty} kA_k^n \left\{|k-u_k|+(1-\alpha)\left|u_k\right|\right\}|a_k| \le 1-\alpha, \quad u_k=1+t+t^2+\cdots+t^{k-1} \quad (2.2)$$

for some $\alpha(0\le\alpha<1)$ then $f(z)\in T^n_{\lambda,\mu}(\alpha,t)$, where

$$A_k^n = [1+(\lambda\mu k+\lambda-\mu)(k-1)]^n.$$

Proof. Noting that $f \in T^n_{\lambda,\mu}(\alpha,t)$ if and only if $zf' \in S^n_{\lambda,\mu}(\alpha,t)$, we can prove Theorem 2.2. □

We now define

$$S^n_{0,\lambda,\mu}(\alpha,t) = \{f \in \mathcal{A} : f \text{ satisfies } (2.1)\}$$

and

$$T^n_{0,\lambda,\mu}(\alpha,t) = \{f \in \mathcal{A} : f \text{ satisfies } (2.2)\}.$$

In view of the above theorems, we see :

Example 2.1. Let us consider a function $f(z)$ given by

$$f(z) = z + (1-\alpha)\left(\frac{\eta\delta_2}{2A^n_2(2-\alpha)}z^2 + \frac{(1-\eta)\delta_3}{A^n_3(7-3\alpha)}z^3\right), \quad 0 \le \eta \le 1,\ |\delta_2| = |\delta_3| = 1.$$

Then for any $t(|t| \le 1,\, t \ne 1)$, $f(z) \in S^n_{0,\lambda,\mu}(\alpha,t) \subset S^n_{\lambda,\mu}(\alpha,t)$.

Example 2.2. Let us consider a function $f(z)$ given by

$$f(z) = z + (1-\alpha)\left(\frac{\eta\delta_2}{4A^n_2(2-\alpha)}z^2 + \frac{(1-\eta)\delta_3}{3A^n_3(7-3\alpha)}z^3\right), \quad 0 \le \eta \le 1,\ |\delta_2| = |\delta_3| = 1.$$

Then for any $t(|t| \le 1,\, t \ne 1)$, $f(z) \in T^n_{0,\lambda,\mu}(\alpha,t) \subset T^n_{\lambda,\mu}(\alpha,t)$.

Remark 2.3. If we take $n = 0$, $t = -1$ in Theorems 2.1 and 2.2 then we get the results given by Cho et al. [6].

3. Coefficient inequalities

Applying Caratheodory function $p(z)$ defined by

$$p(z) = 1 + \sum_{k=1}^{\infty} p_k z^k \tag{3.1}$$

in $\mathcal{U}$, we discuss the coefficient inequalities for the functions f in the subclasses $S^n_{\lambda,\mu}(\alpha,t)$ and $T^n_{\lambda,\mu}(\alpha,t)$.

Theorem 3.1. *If* $f(z) \in S^n_{\lambda,\mu}(\alpha,t)$, *then*

$$|a_k| \leq \frac{\beta}{A^n_k\,|v_k|}\left\{1+\beta\sum_{j=2}^{k-1}\frac{|u_j|}{|v_j|}+\beta^2\sum_{j_2>j_1}^{k-1}\sum_{j_1=2}^{k-2}\frac{|u_{j_1}u_{j_2}|}{|v_{j_1}v_{j_2}|}\right.$$
$$\left.+\beta^3\sum_{j_3>j_2}^{k-1}\sum_{j_2>j_1}^{k-2}\sum_{j_1=2}^{k-3}\frac{|u_{j_1}u_{j_2}u_{j_3}|}{|v_{j_1}v_{j_2}v_{j_3}|}+\cdots+\beta^{k-2}\prod_{j=2}^{k-1}\frac{|u_j|}{|v_j|}\right\},$$

where

$$\beta = 2(1-\alpha),\;\; v_k = k-u_k. \tag{3.2}$$

Proof. We define the function $p(z)$ by

$$p(z)=\frac{1}{1-\alpha}\left(\frac{(1-t)z\left(D^n_{\lambda,\mu}f(z)\right)'}{D^n_{\lambda,\mu}f(z)-D^n_{\lambda,\mu}f(tz)}-\alpha\right)=1+\sum_{k=1}^{\infty}p_kz^k \tag{3.3}$$

for $f(z) \in S^n_{\lambda,\mu}(\alpha,t)$. Then $p(z)$ is a Caratheodory function and satisfies

$$|p_k| \leq 2 \;(k \geq 1). \tag{3.4}$$

Since

$$(1-t)z\left(D^n_{\lambda,\mu}f(z)\right)' = \left[D^n_{\lambda,\mu}f(z)-D^n_{\lambda,\mu}f(tz)\right]\left[\alpha+(1-\alpha)p(z)\right],$$

we have

$$z+\sum_{k=2}^{\infty}kA^n_ka_kz^k=\left(z+\sum_{k=2}^{\infty}kA^n_ku_ka_kz^k\right)\left(1+(1-\alpha)\sum_{k=1}^{\infty}p_kz^k\right)$$

where

$$u_k = 1+t+t^2+\cdots+t^{k-1}.$$

So we get

$$a_k=\frac{1-\alpha}{A^n_k\,(k-u_k)}\left(p_1A^n_{k-1}u_{k-1}a_{k-1}+p_2A^n_{k-2}u_{k-2}a_{k-2}+\dots\right.$$
$$\left.+p_{k-2}A^n_2u_2a_2+p_{k-1}\right). \tag{3.5}$$

From Eq. (3.5), we easily have that

$$|a_2| = \left|\frac{(1-\alpha)}{A^n_2\,(2-u_2)}p_1\right| \leq \frac{2\,(1-\alpha)}{A^n_2\,|2-u_2|},$$

$$|a_3| \leq \frac{2\,(1-\alpha)}{A^n_3\,|3-u_3|}\left(A^n_2\,|u_2a_2|+1\right) \leq \frac{2\,(1-\alpha)}{A^n_3\,|3-u_3|}\left(1+2\,(1-\alpha)\,\frac{|u_2|}{|2-u_2|}\right)$$

and

$$|a_4| \leq \frac{2(1-\alpha)}{A_4^n |4-u_4|} \left\{ 1 + 2(1-\alpha)\left(\frac{|u_2|}{|2-u_2|} + \frac{|u_3|}{|3-u_3|} \right) + 2^2 (1-\alpha)^2 \frac{|u_2 u_3|}{|2-u_2||3-u_3|} \right\}.$$

Thus, using the mathematical induction, we obtain the inequality (3.2). □

Remark 3.2. If we write $\alpha = t = n = 0$ in Theorem 3.1 then we have well known the result

$$f \in S^* \Longrightarrow |a_k| \leq k,$$

where S^* is usual the class of starlike functions.

Remark 3.3. If we take $\alpha = \frac{1}{2}$, $t = 0$, $n = 1$, $\lambda = 1$, $\mu = 0$ in Theorem 3.1 then we obtain

$$|a_k| \leq \frac{1}{k}.$$

Remark 3.4. If we take $\alpha = 0$, $t = -1$, $n = 1$, in Theorem 3.1 then we have

$$|a_k| \leq \frac{1}{A_k}$$

where $A_k = 1 + (\lambda\mu k + \lambda - \mu)(k-1)$ and $\lambda \geqslant \mu \geqslant 0$.

Remark 3.5. If we put $\lambda = \mu = 1$ in Remark 3.4 then we have the following

$$|a_k| \leq \frac{1}{k^2 - k + 1}.$$

Remark 3.6. Equalities in Theorem 3.1 are attended for $f(z)$ given by

$$\frac{(1-t)z\left(D^n_{\lambda,\mu} f(z)\right)'}{D^n_{\lambda,\mu} f(z) - D^n_{\lambda,\mu} f(tz)} = \frac{1 + (1-2\alpha)z}{1-z}.$$

Theorem 3.7. *If* $f(z) \in T^n_{\lambda,\mu}(\alpha, t)$, *then*

$$|a_k| \leq \frac{\beta}{k A_k^n |v_k|} \left\{ 1 + \beta \sum_{j=2}^{k-1} \frac{|u_j|}{|v_j|} + \beta^2 \sum_{j_2 > j_1}^{k-1} \sum_{j_1=2}^{k-2} \frac{|u_{j_1} u_{j_2}|}{|v_{j_1} v_{j_2}|} + \beta^3 \sum_{j_3 > j_2}^{k-1} \sum_{j_2 > j_1}^{k-2} \sum_{j_1=2}^{k-3} \frac{|u_{j_1} u_{j_2} u_{j_3}|}{|v_{j_1} v_{j_2} v_{j_3}|} + \cdots + \beta^{k-2} \prod_{j=2}^{k-1} \frac{|u_j|}{|v_j|} \right\},$$

where

$$\beta = 2(1-\alpha), \;\; v_k = k - u_k. \tag{3.6}$$

4. Distortion inequalities

For the functions $f(z)$ in the classes $S^n_{0,\lambda,\mu}(\alpha,t)$ and $T^n_{0,\lambda,\mu}(\alpha,t)$, we derive

Theorem 4.1. *If* $f(z) \in S^n_{0,\lambda,\mu}(\alpha,t)$, *then*

$$|z| - \sum_{k=2}^{j} |a_k|\,|z|^k - B_j\,|z|^{j+1} \le |f(z)| \le |z| + \sum_{k=2}^{j} |a_k|\,|z|^k + B_j\,|z|^{j+1} \quad (4.1)$$

where

$$B_j = \frac{1-\alpha-\sum\limits_{k=2}^{j} A_k^n\left\{|k-u_k| + (1-\alpha)\,|u_k|\right\}|a_k|}{(j+1-\alpha\,|u_{j+1}|)\,A_{j+1}^n} \quad (j \ge 2). \qquad (4.2)$$

Proof. From the inequality (2.1) we know that

$$\sum_{k=j+1}^{\infty} A_k^n\left\{|k-u_k| + (1-\alpha)\,|u_k|\right\}|a_k| \le 1-\alpha - \sum_{k=2}^{j} A_k^n\left\{|k-u_k| + (1-\alpha)\,|u_k|\right\}|a_k|\,.$$

On the other hand

$$\left\{|k-u_k| + (1-\alpha)\,|u_k|\right\} \ge k-\alpha\,|u_k|\,,$$

and $k-\alpha\,|u_k|$ is monotonically increasing with respect to k. Thus we deduce

$$(j+1-\alpha\,|u_{j+1}|)A_{j+1}^n \sum_{k=j+1}^{\infty} |a_k| \le 1-\alpha-\sum_{k=2}^{j} A_k^n\left\{|k-u_k| + (1-\alpha)\,|u_k|\right\}|a_k|\,,$$

which implies that

$$\sum_{k=j+1}^{\infty} |a_k| \le B_j. \qquad (4.3)$$

Therefore we have the following

$$|f(z)| \le |z| + \sum_{k=2}^{j} |a_k|\,|z|^k + B_j\,|z|^{j+1}$$

and

$$|f(z)| \ge |z| - \sum_{k=2}^{j} |a_k|\,|z|^k - B_j\,|z|^{j+1}\,.$$

This completes the proof of theorem. □

Similarly we have

Theorem 4.2. *If* $f(z) \in T^n_{0,\lambda,\mu}(\alpha, t)$, *then*

$$|z| - \sum_{k=2}^{j} |a_k| |z|^k - C_j |z|^{j+1} \leq |f(z)| \leq |z| + \sum_{k=2}^{j} |a_k| |z|^k + C_j |z|^{j+1} \quad (4.4)$$

and

$$1 - \sum_{k=2}^{j} k |a_k| |z|^{k-1} - D_j |z|^j \leq \left|f'(z)\right| \leq 1 + \sum_{k=2}^{j} k |a_k| |z|^{k-1} + D_j |z|^j \quad (4.5)$$

where

$$C_j = \frac{1 - \alpha - \sum\limits_{k=2}^{j} kA_k^n \{|k - u_k| + (1-\alpha) |u_k|\} |a_k|}{(j+1) \{j + 1 - \alpha |u_{j+1}|\} A_{j+1}^n} \quad (j \geq 2) \quad (4.6)$$

and

$$D_j = \frac{1 - \alpha - \sum\limits_{k=2}^{j} kA_k^n \{|k - u_k| + (1-\alpha) |u_k|\} |a_k|}{\{j + 1 - \alpha |u_{j+1}|\} A_{j+1}^n} \quad (j \geq 2). \quad (4.7)$$

Remark 4.3. If we choice $n = 0$, $t = -1$, $j = 2$ in Theorems 4.1 and 4.2, then we get the results given by Cho et al. [6].

5. Relation between the classes

By the definitions for the classes $S^n_{0,\lambda,\mu}(\alpha, t)$ and $T^n_{0,\lambda,\mu}(\alpha, t)$, evidently we have

$$S^n_{0,\lambda,\mu}(\alpha, t) \subset S^n_{0,\lambda,\mu}(\beta, t) \quad (0 \leq \beta \leq \alpha < 1)$$

and

$$T^n_{0,\lambda,\mu}(\alpha, t) \subset T^n_{0,\lambda,\mu}(\beta, t) \quad (0 \leq \beta \leq \alpha < 1).$$

Let us consider a relation between $S^n_{0,\lambda,\mu}(\alpha, t)$ and $T^n_{0,\lambda,\mu}(\alpha, t)$.

Theorem 5.1. *If* $f(z) \in T^n_{0,\lambda,\mu}(\alpha, t)$, *then* $f(z) \in S^n_{0,\lambda,\mu}(\frac{1+\alpha}{2}, t)$.

Proof. Let $f(z) \in T^n_{0,\lambda,\mu}(\alpha, t)$. Then if $f(z)$ satisfies

$$\frac{|k - u_k| + (1-\beta) |u_k|}{1 - \beta} \leq k \frac{|k - u_k| + (1-\alpha) |u_k|}{1 - \alpha} \quad (5.1)$$

for all $k \geq 2$, then we have that $f(z) \in S^n_{0,\lambda,\mu}(\beta, t)$. From (5.1), we have

$$\beta \leq 1 - \frac{(1-\alpha) |k - u_k|}{k |k - u_k| + (1-\alpha)(k-1) |u_k|}. \quad (5.2)$$

Furthermore, since for all $k \geq 2$

$$\frac{|k-u_k|}{k\,|k-u_k|+(1-\alpha)(k-1)\,|u_k|} \leq \frac{1}{k} \leq \frac{1}{2},$$

we obtain

$$f(z) \in S^n_{0,\lambda,\mu}(\frac{1+\alpha}{2}, t).$$

□

Remark 5.2. Taking $n = 0$ in Theorems 2.1- 5.1, we immediately obtain the results due to Owa et al. [7].

Remark 5.3. If we put $n = 0$, $t = -1$ in Theorems 3.1- 5.1, then we get the results given by Owa et al. [8].

References

[1] D. Răducanu and H. Orhan, *Subclasses of analytic functions defined by a generalized differential operator*, Int. J. Math. Anal., 4 (1) (2010) 1–15.
[2] E. Deniz, H. Orhan, *The Fekete Szegö problem for a generalized subclass of analytic functions*, Kyungpook Math. J. 50 (2010) 37–47.
[3] F. M. Al-Oboudi, *On univalent functions defined by a generalized Salagean operator*, Int. J. Math. Sci., 27 (2004) 1429–1436.
[4] G. S. Sălăgean, *Subclasses of univalent functions*, Comp. Anal.–Proc. 5th Rom.-Finnish Seminar, Bucharest 1981, Part 1, Lec. Notes Math., 1013 (1983) 362–372.
[5] K. Sakaguchi, *On a certain univalent mapping*, J. Math. Soc., Japan 11 (1959) 72–75.
[6] N. E. Cho, O. S. Kwon, S. Owa, *Certain subclasses of Sakaguchi functions*, SEA Bull. Math., 17 (1993) 121–126.
[7] S. Owa, T. Sekine, Rikuo Yamakawa, *On Sakaguchi type functions*, Appl. Math. Comp., 187 (2007) 356–361.
[8] S. Owa, T. Sekine, Rikuo Yamakawa, *Notes on Sakaguchi functions*, RIMS, Kokyuroku, 1414 (2005) 76–82.
[9] B. A. Frasin, *Coefficient inequalities for certain classes of Sakaguchi type functions*, Int. J. Nonlinear Sci., 10 (2) (2010), 206–211.
[10] Y. Polatoğlu, E. Yavuz, *Multivalued Sakaguchi functions*, Gen. Math., 15 (2-3) (2007), 132–140.
[11] B. A. Frasin, M. Darus, *Subordination results on subclasses concerning Sakaguchi functions*, J. Inequal. Appl., 2009, Article ID 574014.
[12] A. W. Goodman, *On uniformly starlike functions*, J. Math. Anal. Appl., 155 (1991), 364–370.
[13] F. Ronning, *On uniform starlikeness and related properties of univalent functions*, Complex Variables, Theory Appl., 24 (1994) 233–239.

(Received: July 21, 2011)

Halit Orhan and Murat Çağlar
Department of Mathematics
Faculty of Science, Ataturk University
25240, Erzurum, Turkey

E–mails: orhanhalit607@gmail.com
mcaglar25@gmail.com

Nihat Yagmur
Department of Mathematics
Faculty of Science and Art
Erzincan University
24100, Erzincan, Turkey
E–mail: nhtyagmur@gmail.com