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NEW NOTIONS VIA b-OPEN SETS

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Abstract. In this paper, we introduce a new class of topological spaces called b-$T_{1/2}$ space in terms of the concept of b-open sets and b-kernal and investigate some of their fundamental properties and also introduce and study some new notions in topological spaces by utilizing b-open sets.

1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis is the study of variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. One of the most well known notions and also an inspiration source is the notion of b-open [1] sets introduced by Andrijevic in 1996. This class is a subset of the class of semi preopen sets [2], that is a subset of a topological space which is contained in the closure of the interior of its closure. Also, a class of b-open sets is a superset of the class of semiopen sets [6], that is a set which is contained in the closure of its interior, and the class of locally dense sets [5] or preopen sets [7], that is a set which is contained in the interior of its closure. Andrijevic studied several fundamental and interesting properties of b-open sets. Among other results, he showed that a rare b-open set is preopen [1]. Recall that a rare set [3] is a set with no interior points. It is well known that for a topological space X, every rare b-open set is semiopen if and only if the interior of a dense subset is dense. In this paper, we will continue the study of new notions involving b-open sets.

2. Preliminaries

Since we shall require the following known definitions, notations and some properties, we recall them in this section. Throughout the present study, a

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space means a topological space. For a subset A of X, $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ represent the closure of A and the interior of A. By a degenerate set we shall mean a set which contains at most one point, that is, it is either a null set or a singleton set.

Definition 2.1. *A subset S of a topological space X is said to be b-open* [1] *if $S \subset \mathrm{Int}(\mathrm{Cl}(S)) \cup \mathrm{Cl}(\mathrm{Int}(S))$. The complement of a b-open set is called a b-closed set* [1].

Definition 2.2. *The intersection of all b-closed (resp. b-open) sets containing A is called the b-closure* [1] *(resp. b-kernal) of A and is denoted by $b\,\mathrm{Cl}(A)$ (resp. b-$ker(A)$).*

In a space, the closure, the derived set, the kernal and the shell of a singleton $\{x\}$ are well-known and denoted by $\mathrm{Cl}(\{x\})$, $d(\{x\})$, $ker(\{x\})$ and $shl(\{x\})$, respectively. Analogously, the b-closure, the b-derived set, the b-kernal and the b-shell of a singleton set $\{x\}$ are denoted by $b\,\mathrm{Cl}(\{x\})$, b-$\mathrm{d}(\{x\})$, b-$ker(\{x\})$ and b-$shl(\{x\})$, respectively.

With this terminology, we have the following,

$$\begin{aligned}&b\,\mathrm{Cl}(\{x\}) = \cap\{F(b\text{-}closed) : x \in F\},\\ &b\text{-}d(\{x\}) = b\,\mathrm{Cl}(\{x\})\backslash\{x\},\\ &b\text{-}ker(\{x\}) = \cap\{G(b\text{-}open) : x \in G\},\\ &b\text{-}shl(\{x\}) = b\text{-}ker(\{x\})\backslash\{x\},\end{aligned}$$

and further,

$$\begin{aligned}&b\,\mathrm{Cl}(\{x\}) = \{y : x \in b\text{-}ker(\{y\})\},\\ &b\text{-}d(\{x\}) = \{y : y \neq x \;\; and \;\; x \in b\text{-}ker(\{y\})\},\\ &b\text{-}ker(\{x\}) = \{y : x \in b\,\mathrm{Cl}(\{y\})\},\\ &b\text{-}shl(\{x\}) = \{y : y \neq x \;\; and \;\; x \in b\,\mathrm{Cl}(\{y\})\}.\end{aligned}$$

Definition 2.3. *A space (X, τ) is said to be b-T_0* [4] *if for each pair of distinct points x, y in X there exists a b-open set A containing x but not y or a b-open set B containing y but not x.*

Definition 2.4. *A space (X, τ) is said to be b-T_1* [4] *if for each pair x, y in X, $x \neq y$, there exists a b-open set G containing x but not y and a b-open set B containing y but not x.*

3. b-$T_{1/2}$-SPACES

We now introduce the following definition

Definition 3.1. *A space (X, τ) is said to be a b-$T_{1/2}$-space, if for all x, y in X, $x \neq y$, b-$ker(\{x\}) \cap b$-$ker(\{y\})$ is either $\emptyset$ or $\{x\}$ or $\{y\}$.*

Theorem 3.2. *Every b-T_1 space is also b-$T_{1/2}$ space.*

Proof. In a b-T_1 space, for each x in X, b-ker$(\{x\})$=$\{x\}$. Hence b-ker$(\{x\}) \cap$ b-ker$(\{y\}) = \emptyset$ for $x \neq y$. Therefore, a b-T_1 space is also a b-$T_{1/2}$ space. □

Remark 3.3. The converse of the Theorem 3.2 need not be true, in general, as seen from the following example.

Example 3.4. Let $X = \{a, b, c\}$ with the topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Then the space X is b-$T_{1/2}$ but not b-T_1.

Theorem 3.5. *Every b-$T_{1/2}$ space is also a b-T_0 space.*

Proof. Let the space X be b-$T_{1/2}$. Then, for any x, y in $X, x \neq y$, b-ker$(\{x\})$ $\cap$ b-ker$(\{y\})$ is either $\emptyset$ or $\{x\}$ or $\{y\}$. Consequently, b-ker$(\{x\}) \neq b$-ker$(\{y\})$ and hence the space X is a b-T_0 space. □

Remark 3.6. The converse of Theorem 3.5 is not true, in general as is evident in the following example.

Example 3.7. Let $X = \{a, b, c\}$ with the topology $\tau = \{\emptyset, \{a\}, X\}$. Then the space X is b-T_0 but not b-$T_{1/2}$.

Definition 3.8. *Let (X, τ) be a topological space and $A \subset X$. The set of all b-limit points of A is said to be the b-derived set of A and is denoted by $bd(A)$.*

Definition 3.9. *A space (X, τ) is said to be a bT_D-space, if for every x in X, b-$d(\{x\})$ is b-closed.*

Theorem 3.10. *Every b-$T_{1/2}$-space is a bT_D space.*

Proof. In a b-$T_{1/2}$-space (X, τ), for any $x \neq y$, b-ker$(\{x\}) \cap b$-ker$(\{y\})$ is either $\emptyset$ or $\{x\}$ or $\{y\}$, and hence $b$shl$(\{x\}) \cap b$shl$(\{y\}) = \emptyset$. We claim that, for each x in X, $b\,\mathrm{Cl}(\{x\})$ is degenerate. For, if $y, z \in b$-d$(\{x\})$ for some $x \in X$, then for y, z in X, $b$shl$(\{y\})$ and $b$shl$(\{z\})$ will not be disjoint. It is sufficient to consider the case when bd$(\{x\}) = \{z\}$. First we observe that the space (X, τ) is b-T_0 and so $b\,\mathrm{Cl}(\{x\}) \neq b\,\mathrm{Cl}(\{z\})$. Therefore, $x \in b\,\mathrm{Cl}(\{z\})$. Now, if for some y other than x, z is such that $y \in b\,\mathrm{Cl}(\{z\})$ $(=b\,\mathrm{Cl}(bd(\{x\})))$, then $y \in b\,\mathrm{Cl}(\{x\})$ and so b-d$(\{x\})$ will not be a singleton set. Therefore, $b\,\mathrm{Cl}(\{z\}) = b\,\mathrm{Cl}(\mathrm{b}{-}d(\{x\})) = b$-d$(\{x\})$. It follows then that every b-$T_{1/2}$-space is bT_D. □

Remark 3.11. The converse of Theorem 3.10 is false. For the space X in Example 3.7 is bT_D-space but is not a b-$T_{1/2}$ space.

Lemma 3.12. *In a space (X, τ), b-ker$(\{x\}) = b$-ker$(b$-ker$(\{x\}))$ for each x in X.*

Proof. For this suppose, $b\text{-}ker(\{x\}) \subset b\text{-}ker(b\text{-}ker(\{x\}))$ is clear. Again for the reverse inclusion, suppose $y \in b\text{-}ker(b\text{-}ker(\{x\}))$. Then $b\text{-}ker(\{x\}) \cap b\,\mathrm{Cl}(\{y\}) \neq \emptyset$, so we can fix, some $z \in b\text{-}ker(\{x\}) \cap b\,\mathrm{Cl}(\{y\})$. Now, $z \in b\,\mathrm{Cl}(\{y\})$ implies $y \in b\text{-}ker(\{z\})$ which together with $z \in b\text{-}ker(\{x\})$ implies that $y \in b\text{-}ker(\{x\})$. □

Lemma 3.13. *In a space (X, τ), $b\text{-}shl(\{x\}) = b\text{-}ker(b\text{-}shl(\{x\}))$ for each x in X.*

Proof. Clearly, $b\text{-}shl(\{x\}) \subset b\text{-}ker(b\text{-}shl(\{x\}))$. To show that $b\text{-}ker(b\text{-shl}(\{x\})) \subset b\text{-}shl(\{x\})$, suppose $y \in b\text{-}ker(b\text{-}shl(\{x\}))$. Then $b\text{-}shl(\{x\}) \cap b\,\mathrm{Cl}(\{y\}) \neq \emptyset$ and then there exists $z \in b\text{-}shl(\{x\}) \cap b\,\mathrm{Cl}(\{y\})$. Therefore, $x \in b\,\mathrm{Cl}(\{z\})$ and $z \in b\,\mathrm{Cl}(\{y\})$. Consequently, $x \in b\,\mathrm{Cl}(\{y\})$ and so $y \in b\text{-}shl(\{x\})$. This proves the result. □

Theorem 3.14. *For a space (X, τ), the following conditions are equivalent:*

(1) *(X, τ) is b-$T_{1/2}$;*
(2) *For all x, y in X, $x \neq y$, either $b\text{-}ker(\{x\}) \cap b\text{-}ker(\{y\}) = \emptyset$ or one of the point has an empty b shell;*
(3) *The space is b-T_0 and $b\text{-}shl(\{x\}) \cap b\text{-}shl(\{y\}) = \emptyset$ for all x, y in X, $x \neq y$;*
(4) *The space is b-T_0 and the b-kernals of the b-shells of any two distinct points are disjoint.*

Proof. (1)⇒(2): In a b-$T_{1/2}$-space (X, τ), for any $x \neq y$ in X, $b\text{-}ker(\{x\}) \cap b\text{-}ker(\{y\})$ is either $\emptyset$ or $\{x\}$ or $\{y\}$. If $b\text{-}ker(\{x\}) \cap b\text{-}ker(\{y\}) = \{x\}$ and so $b\text{-}ker(\{x\}) = \{x\}$ which implies $b\text{-}shl(\{x\}) = \emptyset$.

(2)⇒(1): Straightforward.

(3)⇒(4): Follows from Lemma 3.13.

(1)⇒(3): In a b-$T_{1/2}$-space (X, τ), for $x \neq y$, $b\text{-}ker(\{x\}) \cap b\text{-}ker(\{y\})$ is either $\emptyset$ or $\{x\}$ or $\{y\}$. Then, clearly, $b\text{-}shl(\{x\}) \cap b\text{-}shl(\{y\}) = \emptyset$ and so $b\text{-}ker(\{x\}) \neq b\text{-}ker(\{y\})$ for all $x \neq y$. That is, the space X is b-T_0.

(4)⇒(1): $b\text{-}shl(\{x\}) \cap b\text{-}shl(\{y\}) = \emptyset$ implies that $b\text{-}ker(\{x\}) \cap b\text{-}ker(\{y\})$ is either $\emptyset$ or $\{x\}$ or $\{y\}$ or $\{x, y\}$. But since the space is b-T_0, x, y cannot both be in $b\text{-}ker(\{x\}) \cap b\text{-}ker(\{y\})$. □

4. Weakly b-separated sets

Definition 4.1. *Let (X, τ) be a space and $A \subset X$. Then a set A is said to be weakly b-separated from a set B if there exists a b-open set G such that $A \subset G$ and $G \cap B = \emptyset$ or $A \cap b\,\mathrm{Cl}(B) = \emptyset$.*

Remark 4.2. In view of Lemma 3.2 of [4] and Definition 4.1, we have the following for $x, y \in X$ of a space,

(i) $b\,\mathrm{Cl}(\{x\}) = \{y : y$ is not weakly b-separated from $x\}$ and

(ii) b-$ker(\{x\}) = \{y : y$ is not weakly b-separated from $y\}$.

Definition 4.3. *For any point x of a space (X, τ),*

(i) *the b-derived set of x is denoted by b-$d(\{x\})$ and is defined to be the set b-$d(\{x\}) = b\,\mathrm{Cl}(\{x\}) \setminus \{x\} = \{y : y \neq x$ and y is not weakly b-separated from $x\}$,*

(ii) *the shell of a singleton set $\{x\}$ is denoted by b-$shl(\{x\})$ and is defined to be the set b-$shl(\{x\}) = b$-$ker(\{x\}) \setminus \{x\} = \{y : y \neq x$ and x is not weakly b-separated from $y\}$.*

Definition 4.4. *Let (X, τ) be a space. Then we define*

(i) *b-N-$D = \{x : x \in X$ and b-$d(\{x\}) = \emptyset\}$,*

(ii) *b-N-$shl = \{x : x \in X$ and b-$shl(\{x\}) = \emptyset\}$,*

(iii) *b-$<x> = b\,\mathrm{Cl}(\{x\}) \cap b$-$ker(\{x\})$.*

Theorem 4.5. *Let $x, y \in X$. Then the following conditions hold:*

(i) *$y \in b$-$ker(\{x\})$ if and only if $x \in b\,\mathrm{Cl}(\{y\})$,*

(ii) *$y \in b$-$shl(\{x\})$ if and only if $x \in b$-$d(\{y\})$,*

(iii) *$y \in b\,\mathrm{Cl}(\{x\})$ implies $b\,\mathrm{Cl}(\{y\}) \subset b\,\mathrm{Cl}(\{x\})$ and*

(iv) *$y \in b$-$ker(\{x\})$ implies b-$ker(\{y\}) \subset b$-$ker(\{x\})$.*

Proof. The proof of (i) and (ii) are obvious from Remark 4.2.

(iii) Let $z \in b\,\mathrm{Cl}(\{y\})$. Then z is not weakly b-separated from y. So there exists a b-open set G containing z such that $G \cap \{y\} \neq \emptyset$. Hence $y \in G$ and by assumption $G \cap \{x\} \neq \emptyset$. Hence z is not weakly b-separated from x. So $z \in b\,\mathrm{Cl}(\{x\})$. Therefore, $b\,\mathrm{Cl}(\{y\}) \subset b\,\mathrm{Cl}(\{x\})$.

(iv) Let $z \in b$-$ker(\{y\})$. Then y is not weakly b-separated from z. So $y \in b\,\mathrm{Cl}(\{z\})$. Hence $b\,\mathrm{Cl}(\{y\}) \subset b\,\mathrm{Cl}(\{z\})$. By assumption $y \in b$-$ker(\{x\})$ and then $x \in b\,\mathrm{Cl}(\{y\})$. So $b\,\mathrm{Cl}(\{x\}) \subset b\,\mathrm{Cl}(\{y\})$. Ultimately $b\,\mathrm{Cl}(\{x\}) \subset b\,\mathrm{Cl}(\{z\})$. Hence $x \in b\,\mathrm{Cl}(\{z\})$, that is $z \in b$-$ker(\{x\})$. This shows that b-$ker(\{y\}) \subset b$-$ker(\{x\})$. □

Theorem 4.6. *Let (X, τ) be a topological space and $x, y \in X$. Then,*

(i) *for every $x \in X$, b-$shl(\{x\})$ is degenerate if and only if for all x, $y \in X$, $x \neq y$, b-$d(\{x\}) \cap b$-$d(\{y\}) = \emptyset$;*

(ii) *for every $x \in X$, b-$d(\{x\})$ is degenerate if and only if for all x, $y \in X$, $x \neq y$, b-$shl(\{x\}) \cap b$-$shl(\{y\}) = \emptyset$;*

Proof. Let b-$d(\{x\}) \cap b$-$d(\{y\}) = \emptyset$. Then there exists $z \in X$ such that $z \in b$-$d(\{x\})$ and $z \in b$-$d(\{y\})$. Then $x \neq y \neq z$ and $z \in b\,\mathrm{Cl}(\{x\})$ and $z \in b\,\mathrm{Cl}(\{y\})$, that is, $x, y \in b$-$ker(\{z\})$. Hence, b-$ker(\{z\})$ and so b-$shl(\{z\})$ is not a degenerate set. Conversely, let $x, y \in b$-$shl(\{z\})$. Then we get $x \neq z$, $x \in b$-$ker(\{z\})$ and $y \neq z$ and $y \in b$-$ker(\{z\})$ and hence z is an element if both $b\,\mathrm{Cl}(\{x\})$ and $b\,\mathrm{Cl}(\{y\})$, which is a contradiction. The proof of (ii) is similar and hence omitted. □

Theorem 4.7. *If* $y \in$ *b*-<*x*>*, then* *b*-<*x*> = *b*-<*y*>.

Proof. If $y \in$ *b*-<x>, then by definition, $y \in b\,\mathrm{Cl}(\{x\}) \cap b\text{-}ker(\{x\})$. Hence $y \in b\,\mathrm{Cl}(\{x\})$ and $y \in b\text{-}ker(\{x\})$ and so we have $b\,\mathrm{Cl}(\{y\}) \subset b\,\mathrm{Cl}(\{x\})$ and $b\text{-}ker(\{y\}) \subset b\text{-}ker(\{x\})$. Then $b\,\mathrm{Cl}(\{y\}) \cap b\text{-}ker(\{y\}) \subset b\,\mathrm{Cl}(\{x\}) \cap b\text{-}ker(\{x\})$. Hence *b*-<y> $\subset$ *b*-<x>. The fact that $y \in b\,\mathrm{Cl}(\{x\})$ implies $x \in b\text{-}ker(\{y\})$ and $y \in b\text{-}ker(\{x\})$ implies $x \in b\,\mathrm{Cl}(\{y\})$. Then we have that *b*-<x> $\subset$ *b*-<y> and hence *b*-<x> = *b*-<y>. □

Theorem 4.8. *For all* $x, y \in X$, *b*-<*x*> $\cap$ *b*-<*y*> $= \emptyset$ *or* *b*-<*x*> = *b*-<*y*>.

Proof. If *b*-<x> $\cap$ *b*-<y> $\neq \emptyset$, then there exists $z \in X$ such that $z \in$ *b*-<x> and $z \in$ *b*-<y>. So by Theorem 4.7, *b*-<z> = *b*-<x> = *b*-<y>. □

Now some of the properties of b-T_0, b-T_1 and b-R_0 spaces are derived by means of weakly b-separation.

Theorem 4.9. *A space* (X, τ) *is* b-T_0 *if and only if any of the following conditions hold:*

(i) *For arbitrary* $x, y \in X$, $x \neq y$, *either* x *is weakly* b*-separated from* y *or* y *is weakly* b*-separated from* x.

(ii) $y \in b\,\mathrm{Cl}(\{x\})$ *implies* $x \notin b\,\mathrm{Cl}(\{y\})$.

Proof. (i) Obvious from the definitions. (ii) By the hypothesis, $y \in b\,\mathrm{Cl}(\{x\})$ and so y is not weakly b-separated from x. Since X is b-T_0, x should be weakly b-separated from y, that is, $x \in b\,\mathrm{Cl}(\{y\})$. □

Theorem 4.10. *A space* (X, τ) *is* b-T_0 *if and only if* $(b\,\mathrm{Cl}(\{x\}) \cap \{y\}) \cap (b\,\mathrm{Cl}(\{y\}) \cap \{x\})$ *is degenerate.*

Proof. Suppose X is b-T_0. Then we have any one of the following two cases, x is weakly b-separated from y or y is weakly b-separated from x.

Case (i): If x is weakly b-separated from y, then we have $\{x\} \cap b\,\mathrm{Cl}(\{y\}) = \emptyset$ and $\{y\} \cap b\,\mathrm{Cl}(\{x\})$ is a degenerate set.

Case (ii). If y is weakly b-separated from x, then we have $\{y\} \cap b\,\mathrm{Cl}(\{x\}) = \emptyset$ and $\{x\} \cap b\,\mathrm{Cl}(\{y\})$ is a degenerate set. Hence $(\{x\} \cap b\,\mathrm{Cl}(\{y\})) \cap (\{y\} \cap b\,\mathrm{Cl}(\{x\}))$ is a degenerate set. Conversely, suppose that $(\{x\} \cap b\,\mathrm{Cl}(\{y\})) \cap (\{y\} \cap b\,\mathrm{Cl}(\{x\}))$ is a degenerate set. Then it is either an empty set or a singleton set. If it is an empty set, then there is nothing to prove. If it is a singleton set, it is either $\{x\}$ or $\{y\}$. If it is $\{x\}$, then y is weakly b-separated from x. If it is $\{y\}$, then x is weakly b-separated from y. This shows that (X, τ) is b-T_0. □

Theorem 4.11. *A space* (X, τ) *is* b-T_0 *if and only if* $b\text{-}d(\{x\}) \cap b\text{-}shl(\{x\}) = \emptyset$.

Proof. Suppose that $b\text{-}d(\{x\}) \cap b\text{-}shl(\{x\}) \neq \emptyset$. Then let $z \in b\text{-}d(\{x\})$ and $z \in b\text{-}shl(\{x\})$. Then $z \neq x$, $z \in b\,\mathrm{Cl}(\{x\})$ and $z \in b\text{-}ker(\{x\})$. Hence z is not weakly b-separated from x and also x is not weakly b-separated from z, which is a contradiction. Conversely, let $b\text{-}d(\{x\}) \cap b\text{-}shl(\{x\}) = \emptyset$. Then there exists $z \neq x$, $z \in b\,\mathrm{Cl}(\{x\})$ and $z \notin b\text{-}ker(\{x\})$. Hence if z is not weakly b-separated from x, then x is weakly b-separated from z. □

Corollary 4.12. *If X is b-T_0, then for any $x \in X$, $b\text{-}{<}x{>} = \{x\}$.*

Theorem 4.13. *A space (X, τ) is b-T_1 if and only if one of the following conditions hold:*

(i) *For arbitrary $x, y \in X$, $x \neq y$, x is weakly b-separated from y.*
(ii) *For every $x \in X$, $b\text{-}d(\{x\}) = \emptyset$ or $b\text{-}N\text{-}D(\{x\}) = X$.*
(iii) *For every $x \in X$, $b\text{-}ker(\{x\}) = \{x\}$.*
(iv) *For every $x \in X$, $b\text{-}shl(\{x\}) = \emptyset$ or $b\text{-}N\text{-}shl(\{x\}) = X$.*
(v) *For every $x, y \in X$, $x \neq y$, $b\,\mathrm{Cl}(\{x\}) = b\,\mathrm{Cl}(\{y\}) = \emptyset$.*
(vi) *For every arbitrary $x, y \in X$, $x \neq y$, we have $b\text{-}ker(\{x\}) \cap b\text{-}ker(\{y\}) = \emptyset$.*

Proof. (i), (ii) and (iii) are clear.

(iv) If x is weakly b-separated from y, then for $y \neq x$, we have $y \notin b\,\mathrm{Cl}(\{x\})$, and hence $x \notin b\text{-}ker(\{y\})$. Therefore, we get that $b\text{-}ker(\{y\}) = \{y\}$. The proof of the converse is similar.

(v) As X is b-T_1, $b\,\mathrm{Cl}(\{x\}) = \{x\}$ and $b\,\mathrm{Cl}(\{y\}) = \{y\}$ by Theorem 3.6 of [4]. So, when $x \neq y$, $b\,\mathrm{Cl}(\{x\}) \cap b\,\mathrm{Cl}(\{y\}) = \emptyset$.

(vi) Follows from (v). □

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