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# $P_\omega$-CLOSEDNESS AND ITS GENERALIZATION WITH RESPECT TO A GRILL

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Abstract. In this paper, a new kind of covering axiom *pre*-$\omega$-closedness ($P_\omega$-closedness, for short), stronger than p-closedness due to J. Dontchev et. al. [7] is introduced in terms of pre-$\omega$-open sets [16]. Several characterizations via filter bases and grills [23] along with various properties of this concept are obtained. Grill generalizations of $P_\omega$-closedness and associated concepts have also been investigated.

## 1. Introduction

The notion of $\omega$-open sets introduced by H. Z. Hdeib [8] has been studied extensively in recent years by a good number of researchers. Some of the recent research works related to $\omega$-open sets are found in the papers of H. Z. Hdeib [8, 9], Noiri, Omari and Noorani [16, 17], Omari and Noorani [18, 19], and Zoubi and Nashef [25].

For a long time, topologists have been interested in investigating properties closely related to compactness using different kinds of open-like sets, some of which can be found in papers [2, 3, 4, 5, 6, 7, 11, 14, 20, 24]. Every new invention neighboring compactness, at some stage or other, yields tremendous applications not only within topology itself but also in other branches of applied sciences. Keeping this in mind, a new kind of covering property, $P_\omega$-closedness, stronger than the celebrated concept of p-closedness due to J. Dontchev et. al. [7] is introduced in terms of $\omega$-open sets and allied concepts. We have obtained several properties and investigated various properties along with its grill generalization.

## 2. Prerequisites

Throughout this paper spaces $(X, \tau)$ and $(Y, \sigma)$ (or simply $X$ and $Y$) represent non-empty topological spaces. The closure and the interior of

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a subset $A$ of a space $X$ are denoted by $cl(A)$ and $int(A)$ respectively. Let $A \subset X$. A point $x \in X$ is called a condensation point of A if for each open set $U$ containing $x$, $A \cap U$ is uncountable. A set $A$ is called $\omega$-closed [8] if it contains all of its condensation points and the complement of an $\omega$-closed set is called an $\omega$-open set or equivalently, $A \subset X$ is $\omega$-open if and only if for each $x \in A$ there exists an open set $U$ containing $x$ such that $U - A$ is countable. The set of all $\omega$-open sets of a topological space $(X, \tau)$ is denoted by $\tau_\omega$. It is to be noted that $\tau_\omega$ is a topology on $(X, \tau)$ finer than $\tau$. The interior and the closure of a subset $A$ of a space $X$ with respect to the topology $\tau_\omega$ are denoted by $int^\tau_\omega(A)$ (or simply by $int_\omega(A)$) and $cl^\tau_\omega(A)$ (or simply by $cl_\omega(A)$) respectively. A subset $A$ of a space $X$ is called semi-open [12] (resp. regular open, $\alpha$-open [15], preopen [13], $\beta$-open [1], semi-$\omega$-open [16], $\alpha$-$\omega$-open [16], pre-$\omega$-open [16], $\beta$-$\omega$-open [16]) if $A \subset cl(int(A))$ (resp. $A = int(cl(A))$, $A \subset int(cl(int(A)))$, $A \subset int(cl(A))$, $A \subset cl(int(cl(A)))$, $A \subset cl(int_\omega(A))$, $A \subset int_\omega(cl(int_\omega(A)))$, $A \subset int_\omega(cl(A))$ and $A \subset cl(int_\omega(cl(A))))$. The family of all semi-open (resp. regular open, $\alpha$-open, preopen, $\beta$-open, semi-$\omega$-open $\alpha$-$\omega$-open, pre-$\omega$-open, $\beta$-$\omega$-open) subsets of $(X, \tau)$ is denoted by $SO(X)$ (resp. $RO(X)$, $\tau^\alpha$, $PO(X)$, $\beta O(X)$, $S\omega O(X)$, $\tau^\alpha_\omega$ , $P\omega O(X)$, $\beta\omega O(X)$). It is well known that every preopen set is pre-$\omega$-open. The family of all preopen (resp. preclopen i.e. preclosed as well as preopen) pre-$\omega$-open, regular open) subsets of $X$ containing $x \in X$ is denoted by $PO(X, x)$ (resp. $PCO(X, x)$, $P\omega O(X, x)$, $RO(X, x)$). The complement of a pre-$\omega$-open set is called a pre-$\omega$-closed set. pcl(S) is the intersection of all preclosed subsets of X containing S. $\theta$-preclosure [7] of a subset $S$ of $X$ is the set $pcl_\theta(S) = \{x \in X : pcl(U) \cap S \neq \emptyset$ for all $U \in PO(X, x)\}\}$. If $S = pcl_\theta(S)$, then $S$ is called a $\theta$-preclosed set [7]. The complement of a $\theta$-preclosed set is called a $\theta$-preopen set or equivalently, $S$ is $\theta$-preopen if for each $x \in S$, there exists $U \in PO(X, x)$ such that $pcl(U) \subset S$. A subset $S$ of a space $X$ is called a p-closed set relative to $X$ [7] if every cover of $S$ by preopen sets of $X$ has a finite subfamily whose pre-closures cover $S$. If $S = X$ and $S$ is p-closed set relative to $X$, then $X$ is called a p-closed space. A topological space $X$ is called strongly irresolvable if $S \in PO(X) \Rightarrow S \in SO(X)$. A space $(X, \tau)$ is called strongly compact [10] if every cover of X by preopen sets has a finite subcover.

A filter base $\mathcal{F}$ on a topological space $(X, \tau)$ is said to *pre-$\theta$*-converge [7] to a point $x \in X$ if for each $V \in PO(X, x)$, there exists an $F \in \mathcal{F}$ such that $F \subset pcl(V)$. A filter base $\mathcal{F}$ is said to *pre-$\theta$*-accumulate [7] ( or *pre-$\theta$*-adhere) at $x \in X$ if $pcl(V) \cap F \neq \emptyset$ for every $V \in PO(X, x)$ and every $F \in \mathcal{F}$. The collection of all points of $X$ at which a filter base $\mathcal{F}$ *pre-$\theta$*-adheres is denoted by $p\text{-}\theta\text{-}ad\mathcal{F}$. Thron [23] has defined a grill as a non-empty family $\mathcal{G}$ of non-empty subsets of $X$ satisfying (a) $A \in \mathcal{G}$ and

$A \subset B \Rightarrow B \in \mathcal{G}$ and (b) $A \cup B \in \mathcal{G} \Rightarrow$ either $A \in \mathcal{G}$ or $B \in \mathcal{G}$. Thron [23] has also shown that $\mathcal{F}(\mathcal{G}) = \{A \subset X : A \cap F \neq \emptyset, \forall F \in \mathcal{G}\}$ is a filter on $X$ and that there exists an ultrafilter $\mathcal{F}$ such that $\mathcal{F}(\mathcal{G}) \subset \mathcal{F} \subset \mathcal{G}$. Let $\mathcal{G}$ be a grill on a topological space $(X, \tau)$ and $\phi : P(X) \to P(X)$ be a mapping defined by $\phi(A) = \{x \in X : U \cap A \in \mathcal{G}$, for all $U \in \tau(x)\}$. B. Roy and M. N. Mukherjee [21] proved that $\psi : P(X) \to P(X)$, where $\psi(A) = A \cup \phi(A)$ for all $A \in P(X)$, is a Kuratowski closure operator and hence induces a topology $\tau_G$ on X finer than $\tau$.

## 3. *pre-$\omega$-$\theta$*-OPEN SETS

**Definition 3.1.** *Let $A$ be a subset of a topological space $X$. Then the pre-$\omega$-interior (resp. pre-$\omega$-closure) of $A$ is denoted by $pint^{\tau}_{\omega}(A)$ (resp. $pcl^{\tau}_{\omega}(A)$) and is defined as the set $pint^{\tau}_{\omega}(A) = \cup\{G \subset A : G \in P\omega O(X)\}$ (resp. $pcl^{\tau}_{\omega}(A) = \cap\{G \supset A : X - G \in P\omega O(X)\}$). If no confusion arises, the pre-$\omega$-interior (resp. pre-$\omega$-closure) of $A$ is denoted by $pint_\omega(A)$ (resp. $pcl_\omega(A)$).*

Now we state following theorem.

**Theorem 3.2.** *For subsets $A, B$ of a topological space $X$, the following properties hold:*

(a) *$pcl_\omega(A) \subset pcl(A)$ and $pcl_\omega(A) \subset cl_\omega(A)$.*
(b) *$A \subset B$ implies $pcl_\omega(A) \subset pcl_\omega(B)$ and $pint_\omega(A) \subset pint_\omega(B)$.*
(c) *$pcl_\omega(pcl_\omega(A)) = pcl_\omega(A)$ and $pint_\omega(pint_\omega(A)) = pint_\omega(A)$.*
(d) *$A$ is pre-$\omega$-closed if and only if $pcl_\omega(A) = A$.*
(e) *$A$ is pre-$\omega$-open if and only if $pint_\omega(A) = A$.*
(f) *$pcl_\omega(X - A) = X - pint_\omega(A)$.*
(g) *$pint_\omega(X - A) = X - pcl_\omega(A)$.*

**Remark 3.3.** For a subset $A$ of a topological space, $pcl_\omega(A) \neq pcl(A)$ in general, which is reflected in the following example.

**Example 3.4.** Consider the space $X = \mathbf{N}$ with the topology generated by the base $\mathcal{B} = \{B_n : n \in \mathbf{N}\}$ where $B_n = \{1, n\}$. Then the topology on $X$ is $\tau = \{\emptyset\} \cup \{G \subset \mathbf{N}$: $G$ contains $1\} = PO(X)$. Since $\mathbf{N}$ is countable, $\tau_\omega = P(X) = P\omega O(X)$, where $P(X)$ is the power set of $X$. Let $A$ be a subset of $X$ containing 1. Then $pcl_\omega(A) = A$ and $pcl(A) = \mathbf{N}$.

**Definition 3.5.** *A point $x \in X$ is said to be a pre-$\omega$-$\theta$-accumulation point of a subset $A$ of a topological space $(X, \tau)$ if $pcl_\omega(U) \cap A \neq \emptyset$ for every $U \in PO(X, x)$. The set of all pre-$\omega$-$\theta$-accumulation points of $A$ is called the pre-$\omega$-$\theta$-closure of $A$ and is denoted by $p_\omega cl_\theta(A)$. A subset $A$ of a topological space $(X, \tau)$ is said to be pre-$\omega$-$\theta$-closed if $p_\omega cl_\theta(A) = A$. The complement of a pre-$\omega$-$\theta$-closed set is called a pre-$\omega$-$\theta$-open set.*

**Lemma 3.6.** *A subset $A$ of a space $X$ is pre-$\omega$-$\theta$-open if and only if for each $x \in A$, there exists $V \in PO(X,x)$ such that $pcl_\omega(V) \subset A$.*

*Proof.* Let $A$ be *pre-$\omega$-$\theta$*-open and $x \in A$. Since $X - A$ is *pre-$\omega$-$\theta$*-closed then for $x \in A$, there exists a $V \in PO(X,x)$ such that $pcl_\omega(V) \cap (X - A) = \emptyset$ and thus $pcl_\omega(V) \subset A$.

Conversely, suppose that the condition does not hold. Then there exists an $x \in A$ such that $pcl_\omega(V) \not\subset A$ for all $V \in PO(X,x)$. Thus $pcl_\omega(V) \cap (X - A) \neq \emptyset$ for all $V \in PO(X,x)$ and so $x$ is a *pre-$\omega$-$\theta$*-accumulation point of $X - A$. Hence $X - A$ is not *pre-$\omega$-$\theta$*-closed. □

**Theorem 3.7.** *Let $A$ and $B$ be any subsets of a space $X$. The following properties hold:*

- **(a)** *$\theta$-preclosed sets are pre-$\omega$-$\theta$-closed sets.*
- **(b)** *$p_\omega cl_\theta(A) \subset pcl_\theta(A)$,*
- **(c)** *if $A \subset B$, then $p_\omega cl_\theta(A) \subset p_\omega cl_\theta(B)$,*
- **(d)** *the intersection of an arbitrary family of pre-$\omega$-$\theta$-closed sets is pre-$\omega$-$\theta$-closed in $X$.*

*Proof.* The proof is straightforward and is thus omitted. □

**Remark 3.8.** In a topological space, $p_\omega cl_\theta(A) \neq pcl_\theta(A)$ and a *pre-$\omega$-$\theta$*-closed set may not be $\theta$-preclosed in general, which is reflected in the following example.

**Example 3.9.** In Example 3.4, consider $A = \mathbf{N} - \{1\}$. Then $1 \notin p_\omega cl_\theta(A)$ because $\{1\} \in PO(X,1)$ and $pcl_\omega(\{1\}) \cap A = \emptyset$ but $pcl_\theta(A) = \mathbf{N}$. It is also clear from this example that $A$ is a *pre-$\omega$-$\theta$*-closed set but not a $\theta$-preclosed set in $X$.

**Definition 3.10.** *Let $X$ be a topological space and $A \subset X$. Then $A$ is called a $\omega$-regular (resp. mist-$\omega$-regular) open set if $A = int_\omega(cl(A))$ (resp. $A = int(cl_\omega(A))$). The family of all $\omega$-regular open sets of $X$ is denoted by $R\omega O(X)$.*

**Lemma 3.11.** *The family $R\omega O(X)$ of all $\omega$-regular open sets of $X$ is a base of some topology on $X$.*

*Proof.* Let $x \in X$. Suppose $A$ and $B$ are any two $\omega$-regular open sets of $X$ containing $x$. Consider $C = A \cap B$. Then $C$ is $\omega$-open (so pre-$\omega$-open) containing $x$ and so $int_\omega(cl(C)) \supset C$. On the other hand, $C = int_\omega(cl(A)) \cap int_\omega(cl(B)) = int_\omega(cl(A) \cap cl(B)) \supset int_\omega(cl(A \cap B)) = int_\omega(cl(C))$. Hence the family $R\omega O(X)$ of all $\omega$-regular open sets of $X$ is a base of some topology on $X$. □

In this paper, we consider $\tau_{R_\omega}$ as the topology generated by the base $R\omega O(X)$.

## 4. $P_\omega$-CLOSED SPACES

**Definition 4.1.** *A topological space $X$ is called $P_\omega$-closed (resp. quasi-H-$\omega$-closed) iff every preopen cover of $X$ has a finite subfamily whose pre-$\omega$-closures (resp. $\omega$-closures) cover $X$.*

**Theorem 4.2.** *Let $(X,\tau)$ be quasi-H-$\omega$-closed and strongly irresolvable. Then $(X,\tau)$ is p-closed.*

*Proof.* Let $\{U_\alpha : \alpha \in \Delta\}$ be a preopen cover of $(X,\tau)$. Since $(X,\tau)$ is *quasi-H-$\omega$*-closed, there exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Delta$ such that $X = \cup_{i=1}^{n} cl_\omega(U_{\alpha_i})$. Since $X$ is strongly irresolvable then each $U_\alpha \in SO(X)$ and so $cl_\omega(U_\alpha) \subset cl(U_\alpha) = cl(int(U_\alpha)) = pcl(U_\alpha)$ for each $\alpha \in \Delta$. Thus $(X,\tau)$ is p-closed. □

**Definition 4.3.** *A filter base $\mathcal{F}$ (resp. a grill $\mathcal{G}$) on a topological space $(X,\tau)$ is said to pre-$\omega$-$\theta$-converge to a point $x \in X$ if for each $V \in PO(X,x)$, there exists $F \in \mathcal{F}$ (resp. $F \in \mathcal{G}$) such that $F \subset pcl_\omega(V)$. A filter base $\mathcal{F}$ is said to pre-$\omega$-$\theta$-accumulate (or pre-$\omega$-$\theta$-adhere) at $x \in X$ if $pcl_\omega(V) \cap F \neq \emptyset$ for every $V \in PO(X,x)$ and every $F \in \mathcal{F}$. The collection of all points of $X$ at which the filter base $\mathcal{F}$ pre-$\omega$-$\theta$-adheres is denoted by $p_\omega$-$\theta$-ad$\mathcal{F}$.*

**Theorem 4.4.** *For a topological space $(X,\tau)$ the following conditions are equivalent:*

(a) *$(X,\tau)$ is $P_\omega$-closed,*
(b) *every ultrafilter base pre-$\omega$-$\theta$-converges to some point of $X$,*
(c) *every filter base pre-$\omega$-$\theta$-accumulates at some point of $X$,*
(d) *for every family $\{V_\alpha : \alpha \in \Delta\}$ of preclosed subsets such that $\cap\{V_\alpha : \alpha \in \Delta\} = \emptyset$, there exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Delta$ such that $\cap_{i=1}^{n} pint_\omega(V_{\alpha_i}) = \emptyset$.*

*Proof.* **(a)⇒(b).** Let $(X,\tau)$ be $P_\omega$-closed and $\mathcal{F}$ be an ultrafilter base on $X$ which does not *pre-$\omega$-$\theta$*-converge to any point of $X$. Since $\mathcal{F}$ is an ultrafilter base on $X$, then it can not *pre-$\omega$-$\theta$*-accumulate at any point of $X$. Thus for each $x \in X$, there is an $F_x \in \mathcal{F}$ and a $V_x \in PO(X,x)$ such that $pcl_\omega(V_x) \cap F_x = \emptyset$. Then the family $\{V_x : x \in X\}$ forms a cover of $X$ by preopen subsets. Since $X$ is $P_\omega$-closed, there exists a finite number of points $x_1, x_2, \ldots, x_n \in X$ such that $X = \cup_{i=1}^{n} pcl_\omega(V_{x_i})$. Since $\mathcal{F}$ is a filter base on $X$, there exists an $F' \in \mathcal{F}$ such that $F' \subset \cap_{i=1}^{n}(F_{x_i})$ and thus $F' = \emptyset$ which is a contradiction.

**(b)⇒(c).** Let $\mathcal{F}$ be any filter base on $X$. Then there is an ultrafilter base $\mathcal{F}'$ containing $\mathcal{F}$. By the hypothesis, $\mathcal{F}'$ *pre-$\omega$-$\theta$*-converges to some point $x \in X$. Now consider $V \in PO(X,x)$ and every $F \in \mathcal{F}$. Then there exists an $F' \in \mathcal{F}'$ such that $F' \subset pcl_\omega(V)$ and $F \cap F' \neq \emptyset$. Hence $\emptyset \neq F \cap F' \subset pcl_\omega(V) \cap F$. So the filter base $\mathcal{F}$ *pre-$\omega$-$\theta$*-accumulates at $x \in X$.

**(c)⇒(d).** Let $\{V_\alpha : \alpha \in \Delta\}$ be a family of preclosed subsets of $X$ such that $\cap\{V_\alpha : \alpha \in \Delta\} = \emptyset$. Let $\mathcal{G}$ be the family of all finite subsets of $\Delta$. Suppose $\cap\{pint_\omega(V_\beta), \beta \in \delta\} \neq \emptyset$ for each $\delta \in \mathcal{G}$. Then $\mathcal{F} = \{\cap\{pint_\omega(V_\beta), \beta \in \delta\}, \delta \in \mathcal{G}\}$ is a filter base on $X$. For, if $F_1, F_2 \in \mathcal{F}$, then $F_1 = \cap\{pint_\omega(V_\beta), \beta \in \delta\}$ and $F_2 = \cap\{pint_\omega(V_\gamma), \gamma \in \delta'\}$ for some $\delta, \delta' \in \mathcal{G}$ and so $F_3 = F_1 \cap F_2 = \cap\{pint_\omega(V_\lambda), \lambda \in \delta \cup \delta'\} \in \mathcal{F}$. Then by (c), $\mathcal{F}$ *pre*-$\omega$-$\theta$-accumulates at some point $x$ of $X$. Since $\{X - V_\alpha : \alpha \in \Delta\}$ is a preopen cover of $X$, $x \in X - V_{\alpha_0}$ for some $\alpha_0 \in \Delta$. Let $G = X - V_{\alpha_0}$. Then $G \in PO(X, x)$ and $pint_\omega(V_{\alpha_0}) \in \mathcal{F}$ such that $pcl_\omega(G) \cap pint_\omega(V_{\alpha_0}) = \emptyset$ which is a contradiction.

**(d)⇒(a).** Let $\{U_\alpha : \alpha \in \Delta\}$ be a family of preopen subsets of $X$ covering $X$. Then $\{X - U_\alpha : \alpha \in \Delta\}$ is a family of preclosed subsets of $X$ having empty intersection. Thus by (d), there exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Delta$ such that $\cap_{i=1}^n pint_\omega(X - U_{\alpha_i}) = \emptyset$ i.e. $\cup_{i=1}^n pcl_\omega(U_{\alpha_i}) = X$. So $(X, \tau)$ is $P_\omega$-closed. □

**Theorem 4.5.** *If the topological space $X$ is $P_\omega$-closed, then every pre-$\omega$-$\theta$-open cover of $X$ has a finite subcover.*

*Proof.* Let $X$ be $P_\omega$-closed. Let $\Sigma = \{U_\alpha : \alpha \in \Delta\}$ be a cover of $X$ by *pre*-$\omega$-$\theta$-open sets of $X$. Let $x \in X$ and $x \in U_{\alpha_x}$ for some $\alpha_x \in \Delta$. Then by the Lemma 3.6, there exists a $V_{\alpha_x} \in PO(X, x)$ such that $pcl_\omega(V_{\alpha_x}) \subset U_{\alpha_x}$. Therefore $\Sigma = \{V_{\alpha_x} : x \in X\}$ is a preopen cover of $X$ and hence there exist $x(1), x(2), \ldots, x(n) \in X$ such that $X = \cup_{i=1}^n pcl_\omega(V_{\alpha_{x(i)}})$. So $X = \cup_{i=1}^n U_{\alpha_{x(i)}}$. Hence $\{U_{\alpha_{x(i)}} : x(i) \in X, i = 1, 2, \ldots, n\}$ is the required finite subcover of $\Sigma$. □

It is clear that every $P_\omega$-closed space is p-closed but the converse need not be true.This fact has been established with the following example.

**Example 4.6.** Consider the space $(X, \tau)$ from Example 3.4. Then clearly, $X$ is p-closed because for any $A \in PO(X)$, $pcl(A) = N$. Now observe the cover $\{A_n = \{1, n\} : n \in \mathbf{N}\}$ of $X$ by preopen sets of $X$. Again it is noted that $pcl_\omega(A_n) = \{1, n\}$ and so $\{A_n : n \in \mathbf{N}\}$ is a cover of $X$ by *pre*-$\omega$-$\theta$-open sets of $X$. But it has no finite subcover. Hence by theorem 4.5, $X$ is not $P_\omega$-closed.

**Definition 4.7.** *A topological space $(X, \tau)$ is said to be strongly $p_\omega$-regular if for each point $x \in X$ and each preclosed set $F$ such that $x \notin F$, there exist $V \in PO(X, x)$ and $W \in P\omega O(X)$ such that $F \subset W$ and $V \cap W = \emptyset$.*

**Theorem 4.8.** *A topological space $X$ is strongly $p_\omega$-regular if and only if for each $x \in X$ and for each preopen set $U$ containing $x$, there exists $V \in PO(X, x)$ such that $x \in V \subset pcl_\omega(V) \subset U$.*

*Proof.* Let $X$ be a strongly $p_\omega$-regular space. Suppose $x \in X$ and $U \in PO(X, x)$. Then $F = X - U$ is a preclosed set not containing $x$. Then there exist a $V \in PO(X,x)$ and a $W \in P\omega O(X)$ such that $F \subset W$ and $V \cap W = \emptyset$. So $x \in V \subset pcl_\omega(V) \subset X - W \subset X - F = U$.

Conversely, let $x \in X$ and $F$ be preclosed with $x \notin F$. Then there exists $V \in PO(X,x)$ such that $x \in V \subset pcl_\omega(V) \subset X - F$. Consider $W = X - pcl_\omega(V)$. Then $F \subset W$ and $V \cap W = \emptyset$. So $X$ is strongly $p_\omega$-regular. □

**Theorem 4.9.** *If a topological space $X$ is $P_\omega$-closed and strongly $p_\omega$-regular, then $X$ is strongly compact.*

*Proof.* Let $X$ be $P_\omega$-closed and strongly $p_\omega$-regular. Suppose $\{U_\alpha : \alpha \in \Delta\}$ is a preopen cover of $X$. For each $x \in X$ there exists $\alpha(x) \in \Delta$ such that $U_{\alpha(x)} \in PO(X,x)$ and since $X$ is strongly $p_\omega$-regular by Theorem 4.8, there exists $V_x \in PO(X,x)$ such that $x \in V_x \subset pcl_\omega(V_x) \subset U_{\alpha(x)}$. Then $\{V_x : x \in X\}$ is a family of preopen subsets of $X$ covering $X$ and as $X$ is $P_\omega$-closed, there exist $x_1, x_2, \ldots, x_n \in X$ such that $X = \cup_{i=1}^n pcl_\omega(V_{x_i}) \subset \cup_{i=1}^n U_{\alpha(x_i)}$. Hence $X$ is strongly compact. □

**Definition 4.10.** *A subset $S$ of a topological space $(X,\tau)$ is said to be $P_\omega$-closed relative to $X$ if every cover $\{V_\alpha : \alpha \in \Delta\}$ of $S$ by preopen subsets of $(X,\tau)$ has a finite subfamily whose pre-$\omega$-closures cover $S$.*

**Theorem 4.11.** *For a topological space $(X,\tau)$ and for a subset $S$ of $X$, the following conditions are equivalent:*

(a) *$S$ is $P_\omega$-closed relative to $X$,*
(b) *every ultrafilter base on $X$ which meets $S$ pre-$\omega$-$\theta$-converges to some point of $S$,*
(c) *every filter base on $X$ which meets $S$ pre-$\omega$-$\theta$-accumulates at some point of $S$,*
(d) *for every family $\{V_\alpha : \alpha \in \Delta\}$ of pre-closed subsets of $(X,\tau)$ such that $[\cap\{V_\alpha : \alpha \in \Delta\}] \cap S = \emptyset$, there exists a finite number of indices $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Delta$ such that $[\cap_{i=1}^n pint_\omega(V_{\alpha_i})] \cap S = \emptyset$.*

*Proof.* The proof is quite similar to the proof of the Theorem 4.4 and is thus omitted. □

**Theorem 4.12.** *For a topological space $(X,\tau)$ and for a subset $S$ of $X$ the following two conditions are equivalent:*

(a) *$S$ is $P_\omega$-closed relative to $X$,*
(b) *every grill $\mathcal{G}$ on $X$ containing $S$ pre-$\omega$-$\theta$-converges to some point of $S$.*

*Proof.* (a)⇒(b). Let $\mathcal{G}$ be a grill on $X$ containing $S$ which does not *pre-$\omega$-$\theta$-converge* to any point of $S$. Then for each $x \in S$ and for each $U_x \in PO(X,x)$, $F \not\subset pcl_\omega(U_x)$ for all $F \in \mathcal{G}$. Thus $pcl_\omega(U_x) \notin \mathcal{G}$. Now consider the cover $\{U_x : x \in S\}$ of S. Since $S$ is $P_\omega$-closed relative to $X$, there exist $x_1, x_2, \ldots, x_n \in S$ such that $S \subset \cup_{i=1}^{n} pcl_\omega(U_{x_i})$. Then $\cup_{i=1}^{n} pcl_\omega(U_{x_i}) \in \mathcal{G}$ is a contradiction.

(b)⇒(a). Let S not be $P_\omega$-closed relative to $X$. Then there exists a cover $\{U_\alpha \in PO(X) : \alpha \in \Delta\}$ of $S$ such that $\mathcal{F} = \{S - \cup_{i=1}^{n} pcl_\omega(U_{\alpha_i}) : n < \aleph_0\}$ is a filterbase on $X$. Now consider $\mathcal{G}$, an ultrafilter base containing $\mathcal{F}$. Then $\mathcal{G}$ is a grill containing $S$ and hence by (b), $\mathcal{G}$ converges at some point $x \in S$. Now let $x \in U_\alpha$ for some $\alpha \in \Delta$, there exists $F \in \mathcal{G}$ such that $F \subset pcl_\omega(U_\alpha)$. So $pcl_\omega(U_\alpha) \in \mathcal{G}$. But $S - pcl_\omega(U_\alpha) \in \mathcal{G}$ which is a contradiction. □

**Corollary 4.13.** *A topological space $X$ is $P_\omega$-closed if and only if every grill on $X$ pre-$\omega$-$\theta$-converges to some point of $X$.*

**Theorem 4.14.** *Let $A, B$ be subsets of a space $X$. If $A$ is pre-$\omega$-$\theta$-closed and $B$ is $P_\omega$-closed relative to $X$, then $A \cap B$ is $P_\omega$-closed relative to $X$.*

*Proof.* Let $\{U_\alpha : \alpha \in \Delta\}$ be a cover of $A \cap B$ by preopen subsets of $X$. Since $A$ is *pre-$\omega$-$\theta$*-closed, then for each $x \in B - A$ there exists $V_x \in PO(X, x)$ such that $pcl_\omega(V_x) \cap A = \emptyset$. Then the family $\{U_\alpha : \alpha \in \Delta\} \cup \{V_x : x \in B - A\}$ is a cover of $B$ by preopen subsets of $X$. Since $B$ is $P_\omega$-closed relative to $X$, then there exist a finite number of points $x_1, x_2, \ldots, x_n \in B - A$ and a finite number of indices $\alpha_1, \alpha_2, \ldots, \alpha_m \in \Delta$ such that $B \subset (\cup_{i=1}^{n} pcl_\omega(V_{x_i})) \cup (\cup_{j=1}^{m} pcl_\omega(U_{\alpha_j}))$ and so $A \cap B \subset \cup_{j=1}^{m} pcl_\omega(U_{\alpha_j})$. So $A \cap B$ is $p_\omega$-closed relative to $X$. □

**Corollary 4.15.** *If $X$ is a $P_\omega$-closed space, then every pre-$\omega$-$\theta$-closed subset of $X$ is $P_\omega$-closed relative to $X$.*

**Definition 4.16.** *Let $X$ be a topological space and $A \subset X$. Then mist-$\omega$-boundary of $A$ is the set $\omega$-$Fr(A) = cl_\omega(A) - int(A)$.*

**Definition 4.17.** *A topological space $X$ is called mist-$\omega$-nearly compact if every cover of $X$ by mist-$\omega$-regular open sets has a finite subcover.*

**Definition 4.18.** *For an infinite cardinal number $\kappa$, a topological space $X$ is called $\omega$-$\kappa$-extremely disconnected if the cardinality of the mist-$\omega$-boundary of every mist-$\omega$-regular open set is less than $\kappa$.*

**Theorem 4.19.** *If a topological space $X$ is $P_\omega$-closed and $\omega$-$\aleph_0$-extremely disconnected, then $X$ is mist-$\omega$-nearly compact.*

*Proof.* Let $\Sigma = \{U_\alpha : \alpha \in \Delta\}$ be a cover of $X$ by *mist-$\omega$*-regular open sets of $X$. Since $U_\alpha = int(cl_\omega(U_\alpha)) \subset int(cl(U_\alpha))$ for each $\alpha \in \Delta$, $\Sigma$ is a preopen cover of $X$. Since $X$ is $P_\omega$-closed, there exists a finite set $\Delta_0 \subset \Delta$ such that $X = \cup_{\alpha \in \Delta_0} pcl_\omega(U_\alpha) \subset \cup_{\alpha \in \Delta_0} cl_\omega(U_\alpha)$. Therefore $X = \cup_{\alpha \in \Delta_0} cl_\omega(U_\alpha)$. Now for each $\alpha \in \Delta_0$, $cl_\omega(U_\alpha) = U_\alpha \cup F_\alpha$ where $F_\alpha = cl_\omega(U_\alpha) - U_\alpha =$

$cl_\omega(U_\alpha) - int(cl_\omega(U_\alpha)) = cl_\omega(U_\alpha) - int(int(cl_\omega(U_\alpha))) = cl_\omega(U_\alpha) - int(U_\alpha)$ is the *mist*-$\omega$-boundary of $U_\alpha$. Since $X$ is $\omega$-$\aleph_0$-extremely disconnected, then each $F_\alpha$ is finite and so $F = \cup\{F_\alpha : \alpha \in \Delta_0\}$ is a finite subset of $X$. Thus $\{U_\alpha : \alpha \in \Delta_0\}$ covers $X - F$ and so $X$ is *mist*-$\omega$-nearly compact. □

**Theorem 4.20.** *A $P_\omega$-closed set $G$ relative to a $T_2$ space $X$ is pre-$\omega$-$\theta$-closed if $G$ is pre-$\omega$-open.*

*Proof.* Let $x \in X - G$. Then for each $g \in G$, there exist two open sets $U_g$ and $V_g$ such that $x \in U_g$ and $g \in V_g$ and $U_g \cap V_g = \emptyset$. Then $\{V_g : g \in G\}$ is a cover of $G$ by open (and so preopen) sets of $X$. Since $G$ is a $p_\omega$-closed set relative to $X$, there exist $g_1, g_2, \ldots, g_n \in G$ such that $G \subset \cup_{i=1}^n pcl_\omega(V_{g_i}) \subset pcl_\omega(\cup_{i=1}^n(V_{g_i}))$. Now consider $U = \cap U_{g_i}$ and $V = \cup V_{g_i}$. Then $U \cap V = \emptyset$ and so $pcl_\omega(V) \subset cl_\omega(V) \subset cl(V) \subset X - U$. Therefore $pcl_\omega(V) \cap U = \emptyset$ and so $pint_\omega(pcl_\omega(V)) \cap U = \emptyset$. Hence $pint_\omega(pcl_\omega(V)) \cap pcl_\omega(U) = \emptyset$ and since $G$ is pre-$\omega$-open, $G \cap pcl_\omega(U) = \emptyset$. Thus $G$ is *pre*-$\omega$-$\theta$-closed. □

**Theorem 4.21.** *A quasi-$H$-$\omega$-closed set $G$ relative to a $T_2$ space $X$ is closed in the topological space $(X, \tau_{R_\omega})$ if $G$ is open in $(X, \tau)$.*

*Proof.* Let $x \in X - G$. Then for each $g \in G$, there exist two open sets $U_g$ and $V_g$ such that $x \in U_g$ and $g \in V_g$ and $U_g \cap V_g = \emptyset$. Then $\{V_g : g \in G\}$ is cover of $G$ by open (and so preopen) sets of $X$. Since $G$ is quasi-$H$-$\omega$-closed set relative to $X$, there exist $g_1, g_2, \ldots, g_n \in G$ such that $G \subset \cup_{i=1}^n cl_\omega(V_{g_i}) \subset cl_\omega(\cup_{i=1}^n(V_{g_i}))$. Now consider $U = \cap_{i=1}^n U_{g_i}$ and $V = \cup_{i=1}^n V_{g_i}$. Then $U \cap V = \emptyset$ and so $cl_\omega(V) \subset cl(V) \subset X - U$. Therefore $cl_\omega(V) \cap U = \emptyset$ and so $int(cl_\omega(V)) \cap U = \emptyset$. Hence $int(cl_\omega(V)) \cap cl(U) = \emptyset$ and so $G \cap int_\omega(cl(U)) = \emptyset$. Hence $G$ is closed in $(X, \tau_{R_\omega})$. □

**Definition 4.22.** *A topological space $X$ is called $P_\omega$-closed (resp. quasi-$H$-$\omega$-closed) with respect to a grill $\mathcal{G}$ if every preopen cover $\{V_\alpha : \alpha \in \Delta\}$ of $X$ has a finite subfamily $\{V_{\alpha_i} : \alpha_i \in \Delta, i = 1, 2, \ldots, n\}$ such that $X - \cup_{i=1}^n pcl_\omega(V_{\alpha_i}) \notin \mathcal{G}$ (resp. $X - \cup_{i=1}^n cl_\omega(V_{\alpha_i}) \notin \mathcal{G}$).*

**Theorem 4.23.** *Every $P_\omega$-closed (resp. quasi-$H$-$\omega$-closed) space $X$ is $P_\omega$-closed (resp. quasi-$H$-$\omega$-closed) with respect to any grill $\mathcal{G}$ on $X$.*

*Proof.* Let $(X, \tau)$ be $P_\omega$-closed and $\{U_\alpha : \alpha \in \Delta\}$ be any preopen cover of $X$. Then there exists a finite subset $\Delta_0$ of $\Delta$ such that $X = \cup\{pcl_\omega(U_\alpha) : \alpha \in \Delta_0\}$( resp. $X = \cup\{cl_\omega(U_\alpha) : \alpha \in \Delta_0\}$). Since $\emptyset \notin \mathcal{G}$, $X - \cup\{pcl_\omega(U_\alpha) : \alpha \in \Delta_0\} \notin \mathcal{G}$ (resp. $X - \cup\{cl_\omega(U_\alpha) : \alpha \in \Delta_0\} \notin \mathcal{G}$). □

**Remark 4.24.** It is obvious that if $\mathcal{G}$ is the grill of all nonempty subsets of any topological space $X$, then the concepts of $X$ being $P_\omega$-closed (resp. quasi-$H$-$\omega$-closed) and $P_\omega$-closed (resp. quasi-$H$-$\omega$-closed) with respect to the grill $\mathcal{G}$ are equivalent.

**Theorem 4.25.** *Let $\mathcal{G}$ be a grill on a topological space $(X, \tau)$ containing all nonempty $\omega$-open sets and $X$ be quasi-$H$-$\omega$-closed with respect to the grill $\mathcal{G}$. Then $X$ is quasi-$H$-$\omega$-closed.*

*Proof.* Let $X$ be quasi-$H$-$\omega$-closed with respect to the grill $\mathcal{G}$ and $\{U_\alpha : \alpha \in \Delta\}$ be a cover of $X$ by the preopen sets of $X$. Then there exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Delta$ such that $X - \cup_{i=1}^n cl_\omega(U_{\alpha_i}) \notin \mathcal{G}$. If $int_\omega(X - \cup_{i=1}^n(U_{\alpha_i})) \neq \emptyset$, then $int_\omega(X - \cup_{i=1}^n(U_{\alpha_i})) \in \mathcal{G}$. But $int_\omega(X - \cup_{i=1}^n(U_{\alpha_i})) = X - cl_\omega(\cup_{i=1}^n U_{\alpha_i}) = X - \cup_{i=1}^n cl_\omega(U_{\alpha_i})$. So $X - \cup_{i=1}^n cl_\omega(U_{\alpha_i}) \in \mathcal{G}$, which is a contradiction. Hence $\emptyset = int_\omega(X - \cup_{i=1}^n(U_{\alpha_i})) = X - \cup_{i=1}^n cl_\omega(U_{\alpha_i})$. Thus $X = \cup_{i=1}^n cl_\omega(U_{\alpha_i})$. Therefore $X$ is quasi-$H$-$\omega$-closed. □

**Definition 4.26.** *A topological space $X$ is called weakly $P_\omega$-closed (resp. strongly $P_\omega$-closed, strongly compact) with respect to a grill $\mathcal{G}$ if every preopen (resp. open, preopen) cover $\{V_\alpha : \alpha \in \Delta\}$ of $X$ has a finite subfamily $\{V_{\alpha_i} : \alpha \in \Delta, i = 1, 2, \ldots, n\}$ such that $X - \cup_{i=1}^n int_\omega(V_{\alpha_i}) \notin \mathcal{G}$ (resp. $X - \cup_{i=1}^n pcl_\omega(V_{\alpha_i}) \notin \mathcal{G}$, $X - \cup_{i=1}^n V_{\alpha_i} \notin \mathcal{G}$ ).*

**Definition 4.27.** *A topological space $X$ is called strongly pre-$\omega$-regular with respect to a grill $\mathcal{G}$ if for each $x \in X$ and preclosed set $F$ not containing $x$. Then there exist disjoint sets $U \in PO(X, x)$ and $V \in P\omega O(X)$ such that $F - V \notin \mathcal{G}$.*

**Theorem 4.28.** *A $P_\omega$-closed strongly pre-$\omega$-regular space with respect to a grill $\mathcal{G}$ is strongly compact with respect to the grill $\mathcal{G}$.*

*Proof.* Let $\{U_\alpha : \alpha \in \Delta\}$ be a cover of $X$ by preopen sets of $X$. Then for each $x \in X$, there exists $\alpha(x) \in \Delta$ such that $x \in U_{\alpha(x)}$. Since $X$ is strongly *pre-$\omega$-*regular with respect to the grill $\mathcal{G}$, there exist disjoint sets $P_{\alpha(x)} \in PO(X, x)$ and $Q_{\alpha(x)} \in P\omega O(X)$ such that $(X - U_{\alpha(x)}) - Q_{\alpha(x)} \notin \mathcal{G}$. Here $\{P_{\alpha(x)} : x \in X\}$ is a cover of $X$ by preopen sets of $X$. Since $X$ is $P_\omega$-closed, there exist $x(1), x(2), \ldots, x(n) \in X$ such that $X = \cup_{i=1}^n pcl_\omega(P_{\alpha(x(i))})$.Consider $S_{\alpha(x)} = (X - U_{\alpha(x)}) - Q_{\alpha(x)}$. Here $P_{\alpha(x)} \cap Q_{\alpha(x)} = \emptyset$ implies that $pcl_\omega(P_{\alpha_{(x)}}) \cap Q_{\alpha(x)} = \emptyset$. Now we claim that $pcl_\omega(P_{\alpha(x)}) \subset S_{\alpha(x)} \cup U_{\alpha(x)}$. In fact $q \in pcl_\omega(P_{\alpha_{(x)}})$ but $q \notin U_{\alpha(x)}$ implies that $q \in X - Q_{\alpha(x)}$ and so $q \in ((X - U_{\alpha(x)}) - Q_{\alpha(x)}) = S_{\alpha(x)}$. Thus $X = \cup_{i=1}^n pcl_\omega(P_{\alpha(x(i))}) \subset \cup_{i=1}^n (S_{\alpha(x(i))} \cup U_{\alpha(x(i))})$ and so $X - \cup_{i=1}^n U_{\alpha(x(i))} \subset \cup_{i=1}^n (S_{\alpha(x(i))})$. But for each $i = 1, 2, \ldots, n$, $S_{\alpha(x(i))} \notin \mathcal{G}$ and so $X - \cup_{i=1}^n U_{\alpha(x(i))} \notin \mathcal{G}$. Hence $X$ is strongly compact with respect to the grill $\mathcal{G}$. □

**Theorem 4.29.** *A $T_2$ weakly $P_\omega$-closed space with respect to a grill $\mathcal{G}$ is strongly pre-$\omega$-regular with respect to the grill $\mathcal{G}$.*

*Proof.* Consider $x \in X$ and a preclosed set $F$ not containing $x$. Then for each $y \in F$, there exist disjoint open sets $U_y$ and $V_y$ containing $x$ and $y$

respectively. Therefore $\{V_y : y \in F\} \cup \{X - F\}$ is a preopen cover of $X$. Since $X$ is a weakly $P_\omega$-closed space with respect to the grill $\mathcal{G}$, there exist $y_1, y_2, \dots, y_n \in F$ such that $X - [\cup_{i=1}^n int_\omega(V_{y_i}) \cup int_\omega(X - F))] \notin \mathcal{G}$. Now consider $U = X - pcl(\cup_{i=1}^n (V_{y_i}))$ and $V = \cup_{i=1}^n (V_{y_i})$. Then $U \cap V = \emptyset$, $U \in PO(X, x)$, $V \in PO(X) \subset P\omega O(X)$ and $F - V = F \cap (X - V) = X - [\cup_{i=1}^n (V_{y_i}) \cup (X - F))] \subset X - [\cup_{i=1}^n int_\omega(V_{y_i}) \cup int_\omega(X - F))]$ and so $F - V \notin \mathcal{G}$. Hence $X$ is strongly *pre*-$\omega$-regular with respect to the grill $\mathcal{G}$. □

**Theorem 4.30.** *Let $\mathcal{G}$ be a grill on a topological space $(X, \tau)$ and $(X, \tau)$ be strongly compact with respect to the grill $\mathcal{G}$. Then $(X, \tau_\mathcal{G})$ is strongly $P_\omega$-closed with respect to the grill $\mathcal{G}$.*

*Proof.* Let $(X, \tau)$ be strongly compact with respect to the grill $\mathcal{G}$ and consider $\Sigma$ to be a cover of $X$ by open sets of $(X, \tau_\mathcal{G})$. Then for each $x \in X$, there exists $U_x \in \Sigma$ such that $x \in U_x$. Then there exist a $B_x \in \tau$ and a $V_x \notin \mathcal{G}$ such that $x \in B_x - V_x \subset U_x$. Then $\{B_x : x \in X\}$ is cover of $X$ by open (and so preopen) sets of the space $(X, \tau)$. Since $(X, \tau)$ is strongly compact with respect to the grill $\mathcal{G}$, there exist $x(1), x(2), \dots, x(n) \in X$ such that $X - \cup_{i=1}^n B_{x(i)} \notin \mathcal{G}$. Now $X - \cup_{i=1}^n pcl_\omega^{\tau_\mathcal{G}}(U_{x(i)}) \subset X - \cup_{i=1}^n U_{x(i)} \subset X - \cup_{i=1}^n (B_{x(i)} - V_{x(i)}) \subset (X - \cup_{i=1}^n (B_{x(i)})) \cup (\cup_{i=1}^n (V_{x(i)})) \notin \mathcal{G}$. Hence $(X, \tau_\mathcal{G})$ is strongly $P_\omega$-closed with respect to the grill $\mathcal{G}$. □

**Theorem 4.31.** *Let $\mathcal{G}$ be a grill on $X$. A topological space $(X, \tau)$ is $P_\omega$-closed with respect to the grill $\mathcal{G}$ if and only if every pre-$\omega$-$\theta$-closed subset of $X$ is $P_\omega$-closed with respect to the grill $\mathcal{G}$ and the space $X$.*

*Proof.* Let $(X, \tau)$ be $P_\omega$-closed with respect to the grill $\mathcal{G}$ and $A$ be a *pre*-$\omega$-$\theta$-closed subset of $X$ and let $\Sigma = \{V_\alpha : \alpha \in \Delta\}$ be a cover of $A$ by preopen sets of $X$. Since $X - A$ is a *pre*-$\omega$-$\theta$-open set, for each $x \in X - A$, by the Lemma 3.6, there exists $U_x \in PO(X, x)$ such that $pcl_\omega(U_x) \subset X - A$. Hence $\Sigma \cup \{U_x : x \in X - A\}$ is a preopen cover of $X$ and so there exist $\alpha_1, \alpha_2, \dots, \alpha_n \in \Delta$ and $x_1, x_2, \dots, x_m \in X - A$ such that $X - ((\cup_{i=1}^n pcl_\omega(V_{\alpha_i})) \cup (\cup_{i=1}^m pcl_\omega(U_{x_i}))) \notin \mathcal{G}$. So $A - \cup_{i=1}^n pcl_\omega(V_{\alpha_i}) = A - ((\cup_{i=1}^n pcl_\omega(V_{\alpha_i})) \cup (X - A))) \subset A - ((\cup_{i=1}^n pcl_\omega(V_{\alpha_i})) \cup (\cup_{i=1}^m pcl_\omega(U_{x_i}))) \subset X - ((\cup_{i=1}^n pcl_\omega(V_{\alpha_i})) \cup (\cup_{i=1}^m pcl_\omega(U_{x_i}))) \notin \mathcal{G}$. Therefore $A - \cup_{i=1}^n pcl_\omega(V_{\alpha_i}) \notin \mathcal{G}$ and hence $A$ is $P_\omega$-closed with respect to the grill $\mathcal{G}$ and $X$. Again since $X$ is a *pre*-$\theta$-$\omega$-closed subset of $X$, the converse part of the theorem is obvious. □

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