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SMOOTH SOLUTIONS OF A CLASS OF QUASIELLIPTIC EQUATIONS

ALIK M. NAJAFOV

ABSTRACT. In this paper the smoothness of solutions of one class of quasielliptic equations in the bounded domain $G \subset R^n$ satisfying the flexible λ-horn condition are studied.

Proceeding from the fact that some mixed derivatives $D^\nu f$ may not be estimated by derivative functions contained in the norm of the space $W_p^l(G)$ and on the other hand from the undesirability of using higher order derivatives of a function f, one finds it necessary to consider other types of Sobolev spaces $W_p^l(Q,G)$, that are introduced and studied in [21] with the finite norm

$$\|f\|_{W_p^l(Q,G)} = \sum_{e \subset Q} \sum_{i \in e_n^0 \backslash Q} \left\| D^{l^{e \vee i}} f \right\|_{p,G}$$

where

$$\|f\|_{p,G} = \left\{ \int\limits_R \left[\cdots \left\{ \int\limits_R \left(\int\limits_R \chi_G(x) \, |f(x)|^{p_1} dx_1 \right)^{\frac{p_2}{p_1}} dx_2 \right\}^{\frac{p_3}{p_2}} \cdots \right]^{\frac{p_n}{p_{n-1}}} dx_n \right\}^{\frac{1}{p_n}},$$

χ_G- the characteristic function of the set $G \subset R^n$, $e_n = \{1, 2, \dots, n\}$; $e_n^0 = e_n \cup \{0\}$; $Q-$ be a fixed subset of set e_n; $\emptyset \neq e \subset Q$; $p \in [1, \infty)$; $a \in [0,1]^n$; $\tau \in$ $\in [1, \infty]$; $l \in N^n$; $D^{l^{e \vee i}} f = D_1^{l_1^{e \vee i}} D_2^{l_2^{e \vee i}} \cdots D_n^{l_n^{e \vee i}} f$, $D_j^{l_j^{e \vee i}} = D_j^{l_j}$ for $j \in e \vee i$; $D_j^{l_j^{e \vee i}} = 0$ for $j \in e_n \backslash (e \vee i)$; $j \in e \vee i$-denote that, or $j \in e \subset Q$, or $j = i \in e_n \backslash Q$. Note that these spaces were also defined by A.D. Djabrailov [6] but with the norm of the space $W_p^l(Q,G)$ replacing $D^{l^{e \vee i}} f$ by $D^{l^{e \cup i}} f$ (note, that in case [6] dominate mixed derivatives). Unlike in the paper [6] here the dominant term is either unmixed derivatives, or mixed derivatives, or mixed derivatives and the unmixed derivatives are equal.

At $|Q| = 1$ ($|Q|$ - the number of the set Q) the space $W_p^l(Q,G)$ coincides with the space of Sobolev $W_p^l(G)$, at $Q \equiv e_n$ the space $W_p^l(Q,G)$ coincides

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with the space of Sobolev with dominant mixed derivatives $S_p^l W(G)$, introduced and studied by S.M.Nikolskii [27] with finite norm

$$\|f\|_{S_p^l(Q,G)} = \sum_{e \subseteq e_n} \left\| D^{l^e} f \right\|_{p,G},$$

where $l = (l_1, l_2, \ldots, l_n)$, $l_j \in N$ for $j \in e_n$; $l^e = (l_1^e, l_2^e, \ldots, l_n^e)$, $l_j^e = l$ for $j \in e$, $l_j^e = 0$ for $j \in e_n \setminus e$.

For example, in equation

$$u + u'_x + u'_y + u''_{xy} + u'_z = 0$$

the norm of function u''_{xy} can not be estimated by the norm of space $W^{(1,1,1)}$, but may be estimated by the norm of the Sobolev space with dominating mixed derivatives $S^{(1,1,1)}W$. Therefore we require additional derivatives of the function $u(x, y, z)$.

Let us consider the problems on smoothness of solutions of equations type $(i, i' \in e_n \backslash Q\ )$:

$$\sum_{\substack{\alpha_j \le l_j,\\ \delta_j \le l_j,\\ j \in e \subset Q}} \sum_{\substack{\left|\alpha, \frac{1}{l}\right|_{e_n \backslash Q} \le 1,\\ \left|\delta, \frac{1}{l}\right|_{e_n \backslash Q} \le 1}} D^{\alpha^{e \vee i}} \left(a_{\alpha_{e,i} \delta_{e,i}}(x) D^{\delta^{e \vee i'}} u(x) \right)$$
$$= \sum_{\alpha_j \le l_j, j \in e \subset Q} \sum_{\left|\alpha, \frac{1}{l}\right|_{e_n \backslash Q} \le 1} D^{\alpha^{e \vee i}} f_{\alpha^{e \vee i}}(x), \quad (1)$$

where $\left|\alpha, \frac{1}{l}\right|_{e_n \backslash Q} = \sum\limits_{j \in e_n \backslash Q} \frac{\alpha_j}{l_j}$. Suppose, that the coefficients $a_{\alpha_{e,i} \delta_{e,i}}(x)$ are bounded, measurable functions are in the domain G, $a_{\alpha_{e,i} \delta_{e,i}}(x) = a_{\delta_{e,i} \alpha_{e,i}}(x)$ and

$$\sum_{\substack{\alpha_j = l_j,\\ \delta_j = l_j,\\ j \in e \subset Q}} \sum_{\substack{\left|\alpha, \frac{1}{l}\right|_{e_n \backslash Q} = 1,\\ \left|\delta, \frac{1}{l}\right|_{e_n \backslash Q} = 1}} a_{\alpha_{e,i} \delta_{e,i}}(x) \xi_{\alpha_{e,i}} \xi_{\delta_{e,i}}$$
$$\ge C_0 \sum_{\substack{\alpha_j = l_j,\\ j \in e \subset Q}} \sum_{\left|\alpha, \frac{1}{l}\right|_{e_n \backslash Q} = 1} \left|\xi_{\alpha_{e,i}}\right|^2, \ \xi \in R^n, \ C_0 = \mathrm{const} > 0. \quad (2)$$

We assume that $f_{\alpha^{e \vee i}}(x) \in L_2(G)$ for $\alpha_j < l_j$, $j \in Q$, $\left|\alpha, \frac{1}{l}\right|_{e_n \backslash Q} < 1$; $f_{\alpha^{e \vee i}}(x) \in L_{2,a,æ}(G)$ for $\alpha_j = l_j$, $j \in Q$, $\left|\alpha, \frac{1}{l}\right|_{e_n \backslash Q} = 1$.

A generalized solution to (1) in G is a function $u(x) \in W_2^l(Q, G)$ such that

$$\sum_{\substack{\alpha_j \le l_j,\\ \delta_j \le l_j,\\ j\in e\subset Q}} \sum_{\substack{\left|\alpha,\frac{1}{l}\right|_{e_n\setminus Q}\le 1,\\ \left|\delta,\frac{1}{l}\right|_{e_n\setminus Q}\le 1}} (-1)^{\left|\alpha^{e\vee i}\right|} \int_G a_{\alpha_{e,i}\delta_{e,i}}(x) D^{\delta^{e\vee i'}} u(x) D^{\alpha^{e\vee i}} \upsilon(x)\, dx$$

$$= \sum_{\substack{\alpha_j\le l_j,\\ j\in e\subset Q}} \sum_{\left|\alpha,\frac{1}{l}\right|_{e_n\setminus Q}\le 1} (-1)^{\left|\alpha^{e\vee i}\right|} \int_G f_{\alpha^{e\vee i}}(x) D^{\alpha^{e\vee i}} \upsilon(x)\, dx, \quad (3)$$

for every function $\upsilon(x) \in \overset{\circ}{W}{}_2^{\,l}(Q,G)$. We denote by $\overset{\circ}{W}{}_2^{\,l}(Q,G)$ the supplement of $C_0^\infty(G)$ in the norm $W_2^l(Q,G)$.

Notice that at $|Q| = 1$ ($|Q|$–the number of the set Q) the equation (1) converts into the following

$$\sum_{\substack{\left|\alpha,\frac{1}{l}\right|\le 1,\\ \left|\delta,\frac{1}{l}\right|\le 1}} D^\alpha \left(a_{\alpha\delta}(x) D^\delta u(x)\right) = \sum_{\left|\alpha,\frac{1}{l}\right|\le 1} D^\alpha f_\alpha(x), \quad (4)$$

where $\left|\alpha,\frac{1}{l}\right| = \sum\limits_{j\in e_n} \frac{\alpha_j}{l_j}$, at $Q \equiv e_n$ the equation (1) converts into the following

$$\sum_{\substack{\alpha_j\le l_j,\\ \delta_j\le l_j,\\ j\in e\subset e_n}} D^{\alpha^e} \left(a_{\alpha_e\delta_e}(x) D^{\delta^e} u(x)\right) = \sum_{\substack{\alpha_j\le l_j,\\ j\in e\subset e_n}} D^{\alpha^e} f_{\alpha^e}(x). \quad (5)$$

The problem of the local smoothness of solutions of equations of type (4) was considered by several authors. In [8] the Hölder continuity of solutions of quasielliptic equations with continuous or Hölder continuous coefficients of the leading derivatives is studied. In [1] L_p–estimates for solutions were studied, under the condition that the coefficients of the leading derivatives are infinitely differentiable. In [11] a theorem was proven claiming that the solution belongs to the Hölder class inside the domain, and in [7] local "interior" Hölder estimates were obtained for solutions to a quasielliptic-type equation in the case when the right-hand side satisfies the anisotropic Hölder condition. In this article and [18], [20], [22], [23] we proved theorems stating that the solution belongs to the Hölder class inside the domain, and has a zero boundary Dirichlet condition up to bounds. Notice that in this article, as in [11], we study the Hölder continuity of a solution without any smoothness condition on coefficients. However, observe that unlike [11] here

(1) $\nu \neq 0$;
(2) the Hölder "exponent" is greater than that in [11];
(3) f_α for $\alpha_j = l_j$, $j \in Q$; $\left|\alpha,\frac{1}{l}\right|_{e_n\setminus Q} = 1$ belongs to a broader class, i.e. $f_{\alpha^{e\vee i}} \in L_{2,a,æ}(G)$.

To study partial differential equations it is necessary to study the space of functions of many variables with parameters. For some partial values indexes initially are studied in papers of Morrey [13]-[15], and later on developed in papers [9], [28], [4], [5], [2], [12], [26], [10], [16], [17], [19], [24] and others.

For all $x \in G$, $æ, t \in (0,\infty)^n$; $t_i = t_j$ for $i,j \in e_n \backslash Q$ assuming

$$I_{t^{æ}}(x) = \left\{ y : |y_j - x_j| < \frac{1}{2} t_j^{æ_j}, j \in e_n \right\}, \ \ G_{t^{æ}}(x) = G \cap I_{t^{æ}}(x).$$

Let us consider for $x \in G$ the trajectory

$$\rho(t^{\lambda}, x) = (\rho_1(t_1^{\lambda_1}, x), \rho_2(t_2^{\lambda_2}, x), \ldots, \rho_n(t_n^{\lambda_n}, x)), \ 0 \le t_j \le T_j, \ j \in e_n,$$

where for all $j \in e_n$, $\rho_j(0,x) = 0$, the functions $\rho_j(u_j, x)$ are absolutely continuous with respect to u_j on $[0, T_j^{\lambda_j}]$ and $\left| \rho'_j(u_j, x) \right| \le 1$ for almost all $u_j \in [0, T_j^{\lambda_j}]$, where $\rho'_j(u_j, x) = \frac{\partial}{\partial u_j} \rho_j(u_j, x)$. At $\theta \in (0,1]^n$, $\theta_i = \theta_j$ for $i,j \in e_n \backslash Q$, then the set $V(\lambda, x, \theta) = \bigcup_{0 \le t_j \le T_j, j \in e_n} \left[\rho(t^{\lambda}, x) + t^{\lambda} \theta^{\lambda} I \right]$ we called the set of (A)-condition introduced by the author in [22]. We will suppose that $x +$ $V(\lambda, x, \theta) \subset G$. In the case of, $t_1 = t_2 = \cdots = t_n = t$, $\theta_1 = \theta_2 = \cdots = \theta_n = \theta \in (0,1]$, $V(\lambda, x, \theta)-$is flexible $-\lambda$,-horn introduced by O.V. Besov [3].

Definition. *The Sobolev-Morrey -* $W^l_{p,a,æ,\tau}(Q,G)$ *(* $p \in [1,\infty)^n$, $a \in [0,1]^n$, $\tau \in [1,\infty]$*) type space is the Banach space of locally summable on G functions* f *with finite norm:*

$$\|f\|_{W^l_{p,a,æ,\tau}(Q,G)} = \sum_{e \subset Q} \sum_{i \in e_n^0 \backslash Q} \left\| D^{l^{e \vee i}} f \right\|_{p,a,æ,\tau;G},$$

where

$$\|f\|_{p,a,æ,\tau;G} = \|f\|_{L_{p,a,æ,\tau}(G)}$$
$$= \sup_{x \in G} \left\{ \int_0^{t_0} \left[\prod_{j \in e_n} [t_j]_1^{-\frac{æ_j a_j}{p}} \|f\|_{p, G_{t^{æ}}(x)} \right]^{\tau} \prod_{j \in Q \vee i} \frac{dt_j}{t_j} \right\}^{\frac{1}{\tau}},$$

where $[t_j]_1 = \min\{1, t_j\}, j \in e_n$; $t_0 = (t_{01}, t_{02}, \ldots, t_{0n})-$ *be a fixed positive vector,* $t_{0i} = t_{0j}$ *for* $i,j \in e_n \backslash Q$, $l_0 = 0$. *In the case* $\tau = \infty$, $a = (a, \ldots, a)$, $p = (p, \ldots, p)$, $t = (t, \ldots, t)$ *Sobolev-Morrey space* $W^l_{p,a,æ,\infty}(G) \equiv W^l_{p,a,æ}(G)$ *were introduced and studied by V.P. Il'yin* [12].

Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\lambda_j = 1$ for $j \in Q$, $0 < \lambda_j < \infty$ for $j \in e_n \backslash Q$,

$$\varepsilon_j = \lambda_j l_j - \sum_{j = k \vee j = i \in e_n \backslash Q} \left[\lambda_j \nu_j + (\lambda_j - æ_j a_j) \left(\frac{1}{p} - \frac{1}{q} \right) \right],$$

$$\varepsilon_j^0 = \lambda_j l_j - \sum_{j=k \vee j=i \in e_n \setminus Q} \left[\lambda_j \nu_j + (\lambda_j - æ_j a_j) \frac{1}{p} \right], \quad k \in Q.$$

For the prove of the main results, let us formulate Theorems 1 and 2, which were proved in [24].

Theorem 1. *Let an open set $G \subset R^n$ satisfied of (A)-condition, $1 \le p \le q \le \infty$, $0 < æ_j \le \lambda_j$, $0 < T_j \le 1$, $j \in e_n$, $1 \le \tau_1 \le \tau_2 \le \infty$, $\nu = (\nu_1, \nu_2, \ldots, \nu_n)$, $\nu_j \ge 0-$ be integer, $j \in e_n$, $f \in W^l_{p,a,æ,\tau_1}(Q,G)$ and let $\varepsilon_j > 0$, $j \in e_n$, then*

$$D^\nu : W^l_{p,a,æ,\tau_1}(Q,G) \hookrightarrow L_{q,b,æ,\tau_2}(G),$$

preciously speaking, for the function f the generalized derivatives $D^\nu f$ exists and the inequalities are valid:

$$\|D^\nu f\|_{q,G} \le C_1 \sum_{e \subset Q} \sum_{i \in e_n^0 \setminus Q} \prod_{j \in Q \vee i} T_j^{s_j} \left\| D^{l^{e \vee i}} f \right\|_{p,a,æ,\tau_1;G}, \tag{6}$$

$$\|D^\nu f\|_{q,b,æ,\tau_2;G} \le C_2 \|f\|_{W^l_{p,a,æ,\tau_1}(Q,G)}, \ (p \le q < \infty), \tag{7}$$

where

$$s_j = \begin{cases} \varepsilon_j, & j \in e \vee i, \\ -\nu_j - (1 - æ_j a_j) \left(\frac{1}{p} - \frac{1}{q} \right), & j \in Q \setminus e. \end{cases}$$

In particular, if $\varepsilon_j^0 > 0$, $j \in e_n$, then $D^\nu f$ is continuous on G and the inequality is valid:

$$\sup_{x \in G} |D^\nu f| \le C_1 \sum_{e \subset Q} \sum_{i \in e_n^0 \setminus Q} \prod_{j \in Q \vee i} T_j^{s_j^0} \left\| D^{l^{e \vee i}} f \right\|_{p,a,æ,\tau_1;G}, \tag{8}$$

where

$$s_j^0 = \begin{cases} \varepsilon_j^0, & j \in e \vee i, \\ -\nu_j - (1 - æ_j a_j) \frac{1}{p}, & j \in Q \setminus e, \end{cases}$$

here $T_j \in (0, \min(1, t_{0j})]$, $j \in e_n$, C_1 and C_2 are constants which do not depend on f and a constant C_1 which does not depend on T also.

Theorem 2. *Let the domain G, with parameters p, q, τ_1, τ_2 and vector's $æ, \nu$ satisfy the conditions of Theorem 1. If $\varepsilon_j > 0$, $j \in e_n$, then the derivatives $D^\nu f$ satisfy the Hölder's condition in metric L_q with exponent β^1, that is*

$$\|\Delta(\xi, G) D^\nu f\|_{q,G} \le C \|f\|_{W^l_{p,a,æ,\tau}(Q,G)} \prod_{j \in Q} |\xi_j|^{\beta_j^1} |\xi|^{\beta^1}_{e_n \setminus Q}, \tag{9}$$

where $\beta^1 = (\beta_1^1, \beta_2^1, \ldots, \beta_n^1)$, $\beta_i^1 = \beta_j^1$ for $i, j \in e_n \setminus Q$ and β_j^1–forall number satisfy the following inequalities:

$$0 \le \beta_j^1 \le 1, \qquad \textit{if} \quad \varepsilon_j > 1 \ \textit{for} \ j \in e,$$

$$\begin{array}{ll} 0 \le \beta_j^1 < 1, & \textit{if} \quad \varepsilon_j = 1 \ \textit{for} \ j \in e; 0 \le \beta_j^1 \le 1 \ \textit{for} \ j \in Q \backslash e; \\ 0 \le \beta_j^1 \le \ \varepsilon_j, & \textit{if} \quad \varepsilon_j < 1 \ \textit{for} \ j \in e, \\ 0 \le \beta^1 \le 1, & \textit{if} \quad \dfrac{\varepsilon_0}{\lambda_0} > 1, \\ 0 \le \beta^1 < 1, & \textit{if} \quad \dfrac{\varepsilon_0}{\lambda_0} = 1, \\ 0 \le \beta^1 \le \dfrac{\varepsilon_0}{\lambda_0}, & \textit{if} \quad \dfrac{\varepsilon_0}{\lambda_0} < 1, \end{array}$$

where $\lambda_0 = \max\limits_{j \in e_n \backslash Q} \lambda_j$, $\varepsilon_0 = \min \varepsilon_{Q,i}$, *and*

$$\varepsilon_{Q,i} = \lambda_i l_i - \sum_{j \in e_n \backslash Q} \left[\lambda_j \nu_j + (\lambda_j - æ_j a_j) \left(\frac{1}{p} - \frac{1}{q} \right) \right], \ i \in e_n \backslash Q.$$

If $\varepsilon_j^0 > 0$, $j \in e_n$, *then*

$$\sup_{x \in G} \left| \Delta(\xi, G) D^\nu f \right| \le C \left\| f \right\|_{W^l_{p,a,æ,\tau}(Q,G)} \prod_{j \in Q} \left| \xi_j \right|^{\beta_j^{1,0}} \left| \xi \right|^{\beta^{1,0}}_{e_n \backslash Q}, \tag{10}$$

where $\beta_j^{1,0} (j \in e_n)$ *satisfies the same conditions, that* β_j^1 *with the substitute* ε_j *by* ε_j^0.

Additionally, suppose that G is a bounded domain in R^n.

Theorem 3. *If* $\varepsilon_j = \lambda_j l_j - \sum_{j=k \vee j=i \in e_n \backslash Q} \left(\lambda_j \nu_j + \frac{\lambda_j}{2} \right) > 0, j \in e_n$ $(k \in Q)$, $\lambda_j^{-1} = l_j$, $j \in e_n \backslash Q$, *then any generalized solution of equation* (1) *from* $W_2^l(Q,G)$ *belongs to the space* $C_{\nu+\beta^1}(G^d)$, $\overline{G^d} \subset G$.

Proof of Theorem 3. Existence of the solution of equation is proved with the help of the variational method in [25]. First let all $a_{\alpha_{e,i} \delta_{e,i}}(x)$ except for the ones for which $\alpha_j = l_j$, $j \in Q$, $\left| \alpha, \frac{1}{l} \right|_{e_n \backslash Q} = 1$ and $f_{\alpha^{e \vee i}} = 0, e \subset Q, i = j \in e_n \backslash Q$. Let $d = (d_1, d_2, \ldots, d_n)$–fixed vector, $b = (b_1, b_2, \ldots, b_n)$, $d_i = d_j$, $b_i = b_j$ for $i = j \in e_n \backslash Q$, $x_0 \in G$ and $\sqcap_b(x_0)$ be the parallelepiped in R^n

$$\sqcap_b(x_0) = \left\{ x : |x_j - x_{0j}| < b^{\lambda_j}, j \in e_n \right\},$$

and G^d be a subdomain of the domain G such that $(0 < d_j < 1, \, j \in e_n)$:

$$G^d = \left\{ y : |y_j - x| < d^{\lambda_j}, j \in e_n, x \in \partial G \right\}$$

we shall assume that $b_j \le d_j$, $j \in e_n$. From the variational principle it follows that

$$\int\limits_{\sqcap_b(x_0)} \sum_{\substack{\alpha_j \le l_j,\\ \delta_j \le l_j,\\ j\in e\subset Q}} \sum_{\substack{\left|\alpha,\frac{1}{l}\right|_{e_n\setminus Q}\le 1,\\ \left|\delta,\frac{1}{l}\right|_{e_n\setminus Q}\le 1}} (-1)^{\left|\alpha^{e\vee i}\right|} a_{\alpha_{e,i}\delta_{e,i}}(x) D^{\delta^{e\vee i'}} (\theta(x)(u(x)-p(x))$$

$$\times D^{\alpha^{e\vee i}} (\theta(x)(u(x)-p(x))\,dx \ge \int\limits_{\sqcap_b(x_0)} \sum_{\substack{\alpha_j \le l_j,\\ \delta_j \le l_j,\\ j\in e\subset Q}} \sum_{\substack{\left|\alpha,\frac{1}{l}\right|_{e_n\setminus Q}\le 1,\\ \left|\delta,\frac{1}{l}\right|_{e_n\setminus Q}\le 1}} (-1)^{\left|\alpha^{e\vee i}\right|}$$

$$\times a_{\alpha_{e,i}\delta_{e,i}}(x) D^{\delta^{e\vee i'}} (u(x)-p(x)) D^{\alpha^{e\vee i}} (u(x)-p(x))\,dx$$
$$= A\left(u(x)-p(x), \sqcap_b(x_0)\right), \tag{11}$$

for any $\theta(x) \in C^\infty\left(\sqcap_b(x_0)\right)$ such that $\theta(x) \equiv 1$ in the neighborhood of $\partial \sqcap_b (x_0)$, in any polynomial $p(x)$ of the form

$$p(x) = \sum_{\substack{\alpha_j = l_j,\\ j\in e\subset Q}} \sum_{\left|\alpha,\frac{1}{l}\right|_{e_n\setminus Q}=1} C_\alpha x^\alpha$$

and for an arbitrary solution $u(x)$ of equation (1). Assume in (11)

$$\theta(x) = 1 - \prod_{j\in e_n} \varpi_j\left(\frac{x_j - x_{j0}}{b^{\lambda_j}}\right),$$

where $\varpi_j(t) \in C^\infty(R)$, $\varpi_j(t) = 1$ at $|t| < 2^{-\lambda_j}$, $\varpi_j(t) = 0$ at $|t| \ge 1$, $0 \le \varpi_j(t) \le 1$. It is clear that $\theta(x) \equiv 0$ in $\sqcap_{\frac{b}{2}}(x_0)$, $\theta(x) \equiv 1$ in a neighborhood of $\partial \sqcap_b (x_0)$, where we have taken the coefficients $p(x)$ as

$$\int\limits_{\sqcap_b(x_0)\setminus\sqcap_{\frac{b}{2}}(x_0)} (u(x)-p(x))x^\alpha\,dx = 0.$$

From inequality (11) with the help of (6) we obtain

$$A\left(u(x)-p(x), \sqcap_b(x_0)\right) \le A\left(u(x)-p(x), \sqcap_b(x_0)\setminus \sqcap_{\frac{b}{2}}(x_0)\right)$$
$$+c_1 A\left(u(x)-p(x), \sqcap_b(x_0)\setminus \sqcap_{\frac{b}{2}}(x_0)\right) \le \varsigma A\left(u(x)-p(x), \sqcap_b(x_0)\setminus \sqcap_{\frac{b}{2}}(x_0)\right).$$

Since $A\left(u(x)-p(x), G\right) = A\left(u(x), G\right)$, then

$$A\left(u(x), \sqcap_{\frac{b}{2}}(x_0)\right) \le \left(1 - \frac{1}{\varsigma}\right) A\left(u(x), \sqcap_b(x_0)\right)$$

hence by induction we obtain that

$$A\left(u(x), \sqcap_{\frac{b}{2^k}}(x_0)\right) \leq (1-\frac{1}{\varsigma})^k A\left(u(x), \sqcap_b(x_0)\right).$$

Let $0 < \xi_j < \frac{b}{2^k}$, it follows that $\prod_\xi(x_0) \subset \sqcap_{\frac{b}{2^k}}(x_0)$. Further $k\ln 2 < \ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}$, we take $k = \left[\frac{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}{\ln 2}\right]$, $\omega = 1-\frac{1}{\varsigma}$ then

$$\begin{aligned} A\left(u(x), \sqcap_b(x_0)\right) &\leq \omega^k A\left(u(x), G\right) \\ &< \omega^{\frac{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}{\ln 2}-1} A\left(u(x), G\right) = e^{\frac{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}{\ln 2}\ln\omega - \ln\omega} A\left(u(x), G\right) \\ &= \left(e^{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}\right)^{\left(\frac{\ln\omega}{\ln 2}-\frac{\ln\omega}{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}\right)} A\left(u(x), G\right) \\ &= \left(\prod_{j\in e_n}\frac{b_j}{\xi_j}\right)^{\left(\frac{\ln\omega}{\ln 2}-\frac{\ln\omega}{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}\right)} A\left(u(x), G\right) \\ &= \left(\prod_{j\in e_n}\frac{\xi_j}{b_j}\right)^{\left|\frac{\ln\omega}{\ln 2}-\frac{\ln\omega}{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}\right|} A\left(u(x), G\right) \\ &\leq \left(\prod_{j\in e_n}\frac{\xi_j}{b_j}\right)^{\left|\frac{\ln\omega}{\ln 2}\right|-\left|\frac{\ln\omega}{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}\right|} A\left(u(x), G\right), \end{aligned}$$

for any $x_0 \in G^d$, $b_j \leq d_j$, $j \in e_n$. Denote by $\left|\frac{\ln\omega}{\ln 2}\right| = \sum_{j\in e_n}\eta_j$ and $\left|\frac{\ln\omega}{\ln\prod\limits_{j\in e_n}\frac{b_j}{\xi_j}}\right| = \sum_{j\in e_n}\sigma_j$. It is obvious that $0 < \eta_j = æ_j a_j < 1$, $0 < \sigma_j < 1$, $\xi_j < b_j$, $j \in e_n$; $\xi_i = \xi_j$ for $i, j \in e_n\backslash Q$, $j \in e_n$, then

$$A\left(u(x), \sqcap_\xi(x_0)\right) \leq C\prod_{j\in e_n}\left(\frac{b_j}{\xi_j}\right)^{\eta_j-\sigma_j} A\left(u(x), G\right), \tag{12}$$

and

$$\int_0^1 \cdots \int_0^1 \left[\prod_{j\in e_n} \zeta_j^{-\eta_j} \int_{\sqcap_\zeta(x_0)} u^2(x)dx \right]^{\frac{1}{2}} \prod_{j\in e_n} \frac{d\zeta_j}{\zeta_j} \le C \int_0^1 \cdots \int_0^1 \prod_{j\in e_n} \frac{db_j}{b_j^{1-\frac{1}{2}\sigma_j}}.$$

This means that $u(x) \in L_{2,a,æ,1}(G^d) \subset L_{2,a,æ,\tau}(G^d)$ and also $D^{l^{e\vee i}}u(x) \in L_{2,a,æ,\tau}(G^d)$, for all $e \subset Q$, $i \in e_n^0\backslash Q$, then it follows that $u(x) \in W^l_{2,a,æ,\tau}(Q,G)$. If we check the conditions of Theorems 1 and 2, it turns out that $\varepsilon_j > 0$, $\varepsilon_j^0 > 0$, for $0 < \zeta_j < 1$, $j \in e_n$ and the conditions of Theorems 1 and 2 are satisfied. Thus by Theorem 1 $D^\nu u(x)$ is continuous on G^d and, by Theorem 2 $D^\nu u(x)$ satisfies the Hölder condition, i.e. $u(x) \in C_{\nu+\beta^1}(G^d)$.

Now we consider the nonhomogeneous quasielliptic equation $(i, i' \in e_n^0\backslash Q)$:

$$\sum_{\substack{\alpha_j=l_j,\\ \delta_j=l_j,\\ j\in e\subset Q}} \sum_{\substack{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}=1,\\ \left|\delta,\frac{1}{l}\right|_{e_n\backslash Q}=1}} D^{\alpha^{e\vee i}} \left(a_{\alpha_{e,i}\delta_{e,i}}(x) D^{\delta^{e\vee i'}} u(x) \right) = \sum_{\substack{\alpha_j\le l_j,\\ j\in e\subset Q}} \sum_{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}\le 1} D^{\alpha^{e\vee i}} f_{\alpha^{e\vee i}}(x), \quad (13)$$

where $a_{\alpha_{e,i}\delta_{e,i}}(x)$ satisfies earlier imposed restrictions, inequality (2) is satisfied, $f_{\alpha^{e\vee i}}(x) \in L_2(G)$ for $\alpha_j < l_j$, $j \in Q$, $\left|\alpha, \frac{1}{l}\right|_{e_n\backslash Q} < 1$; $f_{\alpha^{e\vee i}}(x) \in$ $\in L_{2,a,æ}(G)$ for $\alpha_j = l_j$, $j \in Q$, $\left|\alpha, \frac{1}{l}\right|_{e_n\backslash Q} = 1$. We again consider some substitution G^d. Let $x_0 \in G^d$, $b_j \le d_j, j \in e_n$ and u_{b,x_0} be a generalized solution of equation (13) in $\sqcap_b(x_0)$ from the space $\overset{\circ}{W}{}_2^{\,l}(Q, \sqcap_b(x_0))$ i.e.

$$\sum_{\substack{\alpha_j=l_j,\\ \delta_j=l_j,\\ j\in e\subset Q}} \sum_{\substack{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}=1,\\ \left|\delta,\frac{1}{l}\right|_{e_n\backslash Q}=1}} (-1)^{\left|\alpha^{e\vee i}\right|} \int_G a_{\alpha_{e,i}\delta_{e,i}}(x) D^{\delta^{e\vee i'}} u(x) D^{\alpha^{e\vee i}} v(x)\, dx = \sum_{\substack{\alpha_j\le l_j,\\ j\in e\subset Q}} \sum_{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}\le 1} (-1)^{\left|\alpha^{e\vee i}\right|} \int_G f_{\alpha^{e\vee i}}(x) D^{\alpha^{e\vee i}} v(x)dx, \quad (14)$$

Assuming $v(x) \equiv u_{b,x_0}$ in (14) by virtue of (2) we obtain

$$\int_{\sqcap_b(x_0)} \sum_{\substack{\alpha_j\le l_j,\\ j\in e\subset Q}} \sum_{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}\le 1} \left(D^{\alpha^{e\vee i}} u_{b,x_0} \right) dx$$

$$\leq C_1 \sum_{\substack{\alpha_j<l_j,\\ j\in e\subset Q}} \sum_{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}<1} \prod_{j\in Q} b_j^{2l_j-2\nu_j} b^{2-2|\nu,\lambda|_{e_n\backslash Q}} \int\limits_{\sqcap_b(x_0)} f_{\alpha^{e\vee i}}\, dx$$

$$+ \sum_{\substack{\alpha_j=l_j,\\ j\in e\subset Q}} \sum_{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}=1} \int\limits_{\sqcap_b(x_0)} f_{\alpha^{e\vee i}} dx \leq C_2 \prod_{j\in e_n} b_j^{\varsigma_j}, \quad (15)$$

here $\varsigma_i = \varsigma_j = \max\limits_{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}\leq 1} \left\{(2 - 2\left|\nu,\lambda\right|_{e_n\backslash Q}, \left|æ, a\right|_{e_n\backslash Q}\right\}, i, j \in e_n\backslash Q > 0$ and $\varsigma_j = \max\left\{(2l_j - 2\nu_j), æ_j a_j\right\} > 0$, $j \in Q$. C_2 and ς_j do not depend on $u(x)$ and x_0. Since $\overline{u(x)} = u(x) - u_{b,x_0}$ is a solution of equation (1) when the right hand side is zero, therefore for it

$$A\left(\overline{u(x)}, \sqcap_\xi(x_0)\right) \leq C_3 \prod_{j\in e_n} \left(\frac{b_j}{\xi_j}\right)^{\eta_j-\sigma_j} A\left(u(x), G\right), \quad (16)$$

is valid for any $\xi_j < b_j$, $j \in e_n$, if $x_0 \in G^b$. Then from (12) and (13) we obtain

$$A\left(u(x), \sqcap_\xi(x_0)\right) \leq C_4 A\left(\overline{u(x)}, \sqcap_\xi(x_0)\right) + C_5 A\left(u_{b,x_0}, \sqcap_\xi(x_0)\right)$$

$$\leq C_6 \prod_{j\in e_n} \left(\frac{b_j}{\xi_j}\right)^{\eta_j-\sigma_j} A\left(u(x), G\right).$$

Further, we again apply Theorems 1 and 2 and in this case we obtain the required results.

Finally, we consider equation (1) all of whose coefficients are different from zero and exist for small derivatives of the solution. Then we transfer such members to the right hand side of the equation and obtain the required result. The theorem is proved. □

The following theorem on smoothness of solution under the conditions of Theorem 3 holds when the generalized solution satisfies the Dirichlet boundary condition.

Theorem 4. *Let the domain $G \subset R^n$ such that there exists $\varpi = const > 0$ for any point $x_0 \in \partial G$ and the number $\epsilon < 1$ there exists a parallelepiped $\sqcap_{\varpi\epsilon}(x^1)$ such that $\sqcap_{\varpi\epsilon}(x^1) \subset \sqcap_\epsilon(x_0) \cap (R^n\backslash G)$ and $u(x)$ is a solution of equation (1) from the space $\overset{\circ}{W}{}_2^{l}(Q, G)$. If*

$$\varepsilon_j = \lambda_j l_j - \sum_{j=k\vee j=i\in e_n\backslash Q} \left(\lambda_j\nu_j + \frac{\lambda_j}{2}\right) > 0, \quad j \in e_n (k \in Q),$$

then $u(x)$ belongs to the space $C_{\nu+\beta^1}(\overline{G})$.

Proof of Theorem 4. It is sufficiently in this case, to let all $a_{\alpha_{e,i}\delta_{e,i}}(x) = 0$, except for ones for which $\alpha_j = \delta_j = l_j$, $j \in Q$, $\left|\alpha, \frac{1}{l}\right|_{e_n\backslash Q} = \left|\delta, \frac{1}{l}\right|_{e_n\backslash Q} = 1$. Let $x_0 \in \partial G$ and all $f_{\alpha^{e\vee i}} \equiv 0$ in $\sqcap_b(x_0)$ for $e \subset Q$, $i \in e_n\backslash Q$, $u(x) \equiv 0$ outside of G.

From the variational principle it follows that

$$A\left(u(x), \sqcap_b(x_0)\right) \leq A\left(\theta(x)u(x), \sqcap_b(x_0)\right).$$

As $\theta(x) \equiv 0$ in $\sqcap_{\frac{b}{2}}(x_0)$, then

$$A\left(u(x), \sqcap_b(x_0)\right) \leq A\Big(u(x), \sqcap_b(x_0)\backslash \sqcap_{\frac{b}{2}}(x_0)\Big)$$

$$+C_1 \sum_{\substack{\alpha_j<l_j,\\ j\in e\subset Q}} \sum_{\left|\alpha,\frac{1}{l}\right|_{e_n\backslash Q}<1} \prod_{j\in Q} b_j^{-2l_j+2\nu_j} b^{-2+2|\nu,\lambda|_{e_n\backslash Q}} \int\limits_{\sqcap_b(x_0)\backslash\sqcap_{\frac{b}{2}}(x_0)} \Big(D^{\alpha^{e\vee i}}u(x)\Big)^2\, dx.$$

As $u(x)|_{\sqcap_{\varpi b}(x^1)} = 0$, where $\sqcap_{\varpi b}(x^1) \subset \sqcap_b(x_0)\backslash \sqcap_{\frac{b}{2}}(x_0)$, then we have

$$A\left(u(x), \sqcap_b(x_0)\right) \leq r\, A\Big(u(x), \sqcap_{\frac{b}{2}}(x_0)\Big),$$

and hence it follows

$$A\Big(u(x), \sqcap_{\frac{b}{2}}(x_0)\Big) \leq \left(1 - \frac{1}{r}\right) A\left(u(x), \sqcap_b(x_0)\right),$$

consequently

$$A\Big(u(x), \sqcap_{\frac{b}{2^k}}(x_0)\Big) \leq \left(1 - \frac{1}{r}\right)^k A\left(u(x), \sqcap_b(x_0)\right).$$

Therefore

$$A\left(u(x), \sqcap_\xi(x_0)\right) \leq C \prod_{j\in e_n} \left(\frac{b_j}{\xi_j}\right)^{\eta_j-\sigma_j} A\left(u(x), G\right), \tag{17}$$

if $\xi_j < b_j$, $j \in e_n$ for all $x_0 \in \partial G$, $f_{\alpha^{e\vee i}} \equiv 0$ in $\sqcap_b(x_0)$. Let's estimate now $A\left(u(x), \sqcap_\xi(x_0)\right)$ at given $0 < \xi_j < 1$, $j \in e_n$, $\xi_i = \xi_j$ for $i, j \in e_n\backslash Q$, $x_0 \in G$ and $f_{\alpha^{e\vee i}} \neq 0$, for $e \subset Q$, $i \in e_n\backslash Q$. Let us consider two cases:

a) $x_0 \in G^{\sqrt{\xi}}$;

b) $x_0 \notin G^{\sqrt{\xi}}$.

a) In this case for all $\xi_j \leq b_j$, $j \in e_n$ assuming that $b_j = \sqrt{\xi_j}$, $j \in e_n$ we have

$$A\left(u(x), \sqcap_\xi(x_0)\right) \leq C_1 \prod_{j\in e_n} \left(\frac{b_j}{\xi_j}\right)^{\eta_j-\sigma_j} A\left(u(x), G\right)$$

$$+ C_2 \prod_{j\in e_n} b_j^{\varsigma_j} \le C_3 \prod_{j\in e_n} \left(\frac{b_j}{\xi_j}\right)^{\eta_j-\sigma_j} \left(A\left(u(x),G\right)+1\right). \quad (18)$$

b) In this case there exists a point $x^1 \in \partial G$, such that $\sqcap_{2\sqrt{\xi}}(x^1) \supset \sqcap_{\sqrt{\xi}}(x_0)$. Let $b_j > 2\sqrt{\xi_j}$, $j \in e_n$. For all b_j, $j \in e_n$ let us consider $u_{b,x^1}-$ solution of equation (1) in $\sqcap_b(x^1)\cap G$ from the space $\overset{\circ}{W}{}_2^{\,l}\left(Q, \sqcap_b(x^1)\cap G\right)$, for which inequality

$$A\left(u_{b,x^1}, \sqcap_b(x_0)\right) \le C_4 \prod_{j\in e_n} b_j^{\varsigma_j} \quad (19)$$

is valid, if assuming that $u_{b,x^1}\equiv 0$ outside of $\sqcap_b(x^1)\cap G$.

The function $u(x) - u_{b,x^1}$–solution of equation (1) $\sqcap_b(x^1)$, where for all $a_{\alpha_{e,i}\delta_{e,i}}(x) = 0$, except for the ones, for which $\alpha_j = \delta_j = l_j$, $j \in Q$, $\left|\alpha, \frac{1}{l}\right|_{e_n\backslash Q} = \left|\delta, \frac{1}{l}\right|_{e_n\backslash Q} = 1$ and $f_{\alpha^{e\vee i}} \equiv 0$, for $e \subset Q$, $i \in e_n\backslash Q$. From inequalities (17) and (19) we have

$$A\left(u(x), \sqcap_{2\sqrt{\xi}}(x^1)\right) \le C_5 A\left(u - u_{b,x^1}, \sqcap_{2\sqrt{\xi}}(x^1)\right)$$
$$+C_6 A\left(u_{b,x^1}, \sqcap_{2\sqrt{\xi}}(x^1)\right) \le C_7 \prod_{j\in e_n}\left(\frac{b_j}{\xi_j}\right)^{\eta_j-\sigma_j} A\left(u(x),G\right),$$

$$A\left(u(x), \sqcap_{\xi}(x_0)\right) \le A\left(u(x), \sqcap_{\sqrt{\xi}}(x^1)\right)$$
$$\le A\left(u(x), \sqcap_{2\sqrt{\xi}}(x^1)\right) \le C_8 \prod_{j\in e_n}\left(\frac{b_j}{\xi_j}\right)^{\eta_j-\sigma_j} A\left(u(x),G\right),$$

consequently

$$\int_0^1 \cdots \int_0^1 \left[\prod_{j\in e_n} \zeta_j^{-\eta_j} \int_{\sqcap_\zeta(x_0)} u^2(x)\,dx\right]^{\frac{1}{2}} \prod_{j\in e_n} \frac{d\zeta_j}{\zeta_j} \le C \int_0^1 \cdots \int_0^1 \prod_{j\in e_n} \frac{db_j}{b_j^{1-\frac{1}{2}\sigma_j}}.$$

This implies that $u(x) \in L_{2,a,æ,1}(G^d) \subset L_{2,a,æ,\tau}(G^d)$ and also $D^{l^{e\vee i}}u(x) \in L_{2,a,æ,\tau}(G^d)$, for all $e \subset Q$, $i \in e_n^0\backslash Q$, then it follows that $u(x) \in W^l_{2,a,æ,\tau}$ (Q,G). Then in this case the conditions in Theorems 1 and 2 are satisfied. Thus by Theorems 1 and 2 it follows that $u(x) \in C_{\nu+\beta^1}(\overline{G})$. The theorem is proved. □

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Azerbaijan University of
Architecture and Construction
Az 1073, Baku, Azerbaijan
Ayna Sultanova str., 5
E–mail: nadjafov@rambler.ru