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ON θ-b–IRRESOLUTE FUNCTIONS

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ABSTRACT. The concept of b-open sets was introduced by Andrijevic. The aim of this paper is to introduce and characterize θ-b-irresolute functions by using b-open sets.

1. INTRODUCTION

In 1965, Njastad [7] initiated the study of so called α-open sets. This notion has been studied extensively in recent years by many topologists. As a generalization of open sets, b-open sets were introduced and studied by Andrijevic. This notions was further studied by Ekici [3, 4, 5], Park [8] and Caldas et al [2]. In this paper, we will continue the study of related functions with b-open [1] sets. We introduce and characterize the concepts of θ-b-irresolute functions and relationships between strongly b-irresolute functions and graphs are investigated.

Throughout this paper, X and Y always refer to topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ denote the closure of A and interior of A in X, respectively. A subset A of X is said to be α-open [7] (resp. b-open [1]) if $A \subset \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$ (resp. $A \subset \mathrm{cl}(\int(A)) \cup \mathrm{int}(\mathrm{cl}(A))$). The complement of b-open set is called b-closed [1]. The intersection of all b-closed sets of X containing A is called the b-closure [1] of A and is denoted by $b\,\mathrm{cl}(A)$. A set A is b-closed [8] if and only if $b\,\mathrm{cl}(A) = A$. The union of all b-open sets of X contained in A is called the b-interior [1] of A and is denoted by $b\,\mathrm{int}(A)$. A set A is said to be b-regular [8] if it is b-open and b-closed. The family of all α-open (resp. b-open, b-closed, b-regular) sets of X is denoted by $\alpha O(X)$ (resp. $BO(X)$, $BC(X)$, $BR(X)$). We will set $BO(X, x)$ $= \{V \in BO(X) | x \in V\}$ for $x \in X$.

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2. Preliminaries

A point x of X is called a b-θ-cluster [8] point of $S \subset X$ if $b\,\mathrm{cl}(U) \cap S \neq \varnothing$ for every $U \in BO(X, x)$. The set of all b-θ-cluster points of S is called the b-θ-closure of S and is denoted by $b\,\mathrm{cl}_\theta(S)$. A subset S is said to be b-θ-closed [8] if and only if $S = b\,\mathrm{cl}_\theta(S)$. The complement of a b-θ-closed set is said to be b-θ-open [8].

Theorem 2.1. [8] *Let A be a subset of a topological space X. Then,*

(i) *$A \in BO(X)$ if and only if $b\,\mathrm{cl}(A) \in BR(X)$.*
(ii) *$A \in BC(X)$ if and only if $b\,\mathrm{int}(A) \in BR(X)$.*

Theorem 2.2. [8] *For a subset A of a topological space X, the following properties hold:*

(i) *If $A \in BO(X)$, then $b\,\mathrm{cl}(A) = b\,\mathrm{cl}_\theta(A)$,*
(ii) *$A \in BR(X)$ if and only if A is b-θ-open and b-θ-closed.*

Definition 2.3. [8] *A topological space X is said to be b-regular if for each closed set F and each $x \notin F$, there exist disjoint b-open sets U and V such that $x \in U$ and $F \subset V$.*

Theorem 2.4. [8] *For a topological space X, the following properties are equivalent:*

(i) *X is b-regular;*
(ii) *For each open set U and each $x \in U$, there exists $V \in BO(X)$ such that $x \in V \subset b\,\mathrm{cl}(V) \subset U$;*
(iii) *For each open set U and each $x \in U$, there exists $V \in BR(X)$ such that $x \in V \subset U$.*

Definition 2.5. *A function $f : X \to Y$ is said to be b-irresolute* [6] *if $f^{-1}(V) \in BO(X)$ for every $V \in BO(Y)$.*

Definition 2.6. *A function $f : X \to Y$ is said to be weakly b-irresolute* [10] *if for each $x \in X$ and each $V \in BO(Y, f(x))$, there exists a $U \in BO(X, x)$ such that $f(U) \subset b\,\mathrm{cl}(V)$.*

3. θ-b-irresolute functions

We introduce the following definition

Definition 3.1. *A function $f : X \to Y$ is said to be θ-b-irresolute if for each $x \in X$ and each $V \in BO(Y, f(x))$, there exists $U \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$.*

Clearly, every b-irresolute function is θ-b-irresolute and every θ-b-irresolute function is weakly b-irresolute. But the converses are not true as shown by the following example.

Example 3.2. Let $X = \{a, b, c\} = Y$, $\tau = \{\varnothing, \{a\}, X\}$ and $\sigma = \{\varnothing, \{b\}, Y\}$. Then the identity map $f : (X, \tau) \to (Y, \sigma)$ is θ-b-irresolute but not b-irresolute.

Theorem 3.3. *For a function $f : X \to Y$ the following properties are equivalent:*

(i) *f is θ-b-irresolute;*
(ii) *$b\,\mathrm{cl}_\theta(f^{-1}(B)) \subset f^{-1}(b\,\mathrm{cl}_\theta(B))$ for every subset B of Y;*
(iii) *$f(b\,\mathrm{cl}_\theta(A)) \subset b\,\mathrm{cl}_\theta(f(A))$ for every subset A of X.*

Proof. **(i)⇒(ii):** Let B be any subset of Y. Suppose that $x \notin f^{-1}(b\,\mathrm{cl}_\theta(B))$. Then $f(x) \notin b\,\mathrm{cl}_\theta(B)$ and there exits $V \in BO(Y, f(x))$ such that $b\,\mathrm{cl}(V) \cap B = \varnothing$. Since f is θ-b-irresolute, there exists $U \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$. Therefore, we have $f(b\,\mathrm{cl}(U)) \cap B = \varnothing$ and $b\,\mathrm{cl}(U) \cap f^{-1}(B) = \varnothing$. This shows that $x \notin b\,\mathrm{cl}_\theta(f^{-1}(B))$. Hence, we obtain $b\,\mathrm{cl}_\theta(f^{-1}(B)) \subset f^{-1}(b\,\mathrm{cl}_\theta(B))$.
(ii)⇒(iii): Let A be any subset of X. Then we have $b\,\mathrm{cl}_\theta(A) \subset b\,\mathrm{cl}_\theta(f^{-1}(f(A))) \subset f^{-1}(b\,\mathrm{cl}_\theta(f(A)))$ and hence $f(b\,\mathrm{cl}_\theta(A)) \subset b\,\mathrm{cl}_\theta(f(A))$.
(iii)⇒(ii): Let B be a subset of Y. We have $f(b\,\mathrm{cl}_\theta(f^{-1}(B))) \subset b\,\mathrm{cl}_\theta(f(f^{-1}(B))) \subset b\,\mathrm{cl}_\theta(B)$ and hence $b\,\mathrm{cl}_\theta(f^{-1}(B)) \subset f^{-1}(b\,\mathrm{cl}_\theta(B))$.
(ii)⇒(i): Let $x \in X$ and $V \in BO(Y, f(x))$. Then we have $b\,\mathrm{cl}(V) \cap (Y - b\,\mathrm{cl}(V)) = \varnothing$ and $f(x) \notin b\,\mathrm{cl}_\theta(Y - b\,\mathrm{cl}(V))$. Hence, $x \notin f^{-1}(b\,\mathrm{cl}_\theta(Y - b\,\mathrm{cl}(V)))$ and $x \notin b\,\mathrm{cl}_\theta(f^{-1}(Y - b\,\mathrm{cl}(V)))$. There exists $U \in BO(X, x)$ such that $b\,\mathrm{cl}(U) \cap f^{-1}(Y - b\,\mathrm{cl}(V)) = \varnothing$ and hence $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$. This shows that f is θ-b-irresolute. □

A point x of X is called a b-θ-interior point of A if there exists a b-open set U containing x such that $b\,\mathrm{cl}(U) \subset A$. The set of all b-θ-interior points of A is said to be the b-θ-interior of A [8], denoted by $b\,\mathrm{int}_\theta(A)$.

Theorem 3.4. *For a function $f : X \to Y$ the following properties are equivalent:*

(i) *f is b-θ-irresolute;*
(ii) *$f^{-1}(V) \subset b\,\mathrm{int}_\theta(f^{-1}(b\,\mathrm{cl}(V)))$ for every $V \in BO(Y)$.*
(iii) *$b\,\mathrm{cl}_\theta(f^{-1}(V)) \subset f^{-1}(b\,\mathrm{cl}(V))$ for every $V \in BO(Y)$.*

Proof. **(i)⇒(ii):** Suppose that $V \in BO(Y)$ and $x \in f^{-1}(V)$. Then $f(x) \in V$ and there exists $U \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(U)$. Therefore, $x \in U \subset b\,\mathrm{cl}(U) \subset f^{-1}(b\,\mathrm{cl}(V))$. This shows that $x \in b\,\mathrm{int}_\theta(f^{-1}(b\,\mathrm{cl}(V)))$. This shows that $f^{-1}(V) \subset b\,\mathrm{int}_\theta(f^{-1}(b\,\mathrm{cl}(V)))$.
(ii)⇒(iii): Suppose that $V \in BO(Y)$ and $x \notin f^{-1}(b\,\mathrm{cl}(V))$. Then $f(x) \notin b\,\mathrm{cl}(V)$ and there exists $U \in BO(Y, f(x))$ such that $U \cap V = \varnothing$ and hence $b\,\mathrm{cl}(U) \cap V = \varnothing$. Therefore, we have $f^{-1}(b\,\mathrm{cl}(U)) \cap f^{-1}(V) = \varnothing$. Since $x \in f^{-1}(U)$, by (ii), $x \in b\,\mathrm{int}_\theta(f^{-1}(b\,\mathrm{cl}(U)))$. There exists $W \in BO(X, x)$

such that $b\,\mathrm{cl}(W) \subset f^{-1}(b\,\mathrm{cl}(U))$. Thus, we have $b\,\mathrm{cl}(W) \cap f^{-1}(V) = \varnothing$ and hence $x \notin b\,\mathrm{cl}_\theta(f^{-1}(V))$. This shows that $b\,\mathrm{cl}_\theta(f^{-1}(V)) \subset f^{-1}(b\,\mathrm{cl}(V))$.
(iii)⇒(i): Suppose that $x \in X$ and $V \in BO(Y, f(x))$. Then, $V \cap (Y - b\,\mathrm{cl}(V)) = \varnothing$ and $f(x) \notin b\,\mathrm{cl}(Y - bcl(V))$. Therefore, $x \notin f^{-1}(b\,\mathrm{cl}(Y - blc(V)))$ and by (iii), $x \notin b\,\mathrm{cl}_\theta(f^{-1}(Y - b\,\mathrm{cl}(V)))$. There exists $U \in BO(X, x)$ such that $b\,\mathrm{cl}(U) \cap f^{-1}(Y - b\,\mathrm{cl}(V)) = \varnothing$. Therefore, we obtain $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$. This shows that f is θ-b-irresolute. □

A function $f : X \to Y$ is said to be strongly b-irresolte [9] if for each point $x \in X$ and each $V \in BO(Y, f(x))$, there exists a $U \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U)) \subset V$.

Theorem 3.5. *Let Y be a b-regular space. Then for a function $f : X \to Y$ the following properties are equivalent:*

(i) *f is strongly b-irresolute;*
(ii) *f is b-irresolute;*
(iii) *f is θ-b-irresolute.*

Proof. **(i)⇒(ii):** This is obvious.
(ii)⇒(iii): Suppose that $x \in X$ and $V \in BO(Y, f(x))$. Since f is b-irresolute, $f^{-1}(V)$ is b-open and $f^{-1}(b\,\mathrm{cl}(V))$ is b-closed in X. Now, put $U = f^{-1}(V)$. Then we have $U \in BO(X, x)$ and $b\,\mathrm{cl}(U) \subset f^{-1}(b\,\mathrm{cl}(V))$. Therefore, we obtain $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$. This shows that f is θ-b-irresolute.
(iii)⇒(i): Suppose that $x \in X$ and $V \in BO(Y, f(x))$. Since Y is b-regular, there exists $W \in BO(Y)$ such that $f(x) \in W \subset b\,\mathrm{cl}(W) \subset V$. Since f is θ-b-irresolute, there exists $U \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(W) \subset V$. This shows that f is strongly b-irresolute. □

Theorem 3.6. *Let X be a b-regular space. Then $f : X \to Y$ is b-θ-irresolute if and only if it is weakly b-irresolute.*

Proof. Suppose that f is weakly b-irresolute. Let $x \in X$ and $V \in BO(Y, f(x))$. Then, there exists $U \in BO(X, x)$ such that $f(U) \subset b\,\mathrm{cl}(V)$. Since X is b-regular, there exists $U_0 \in BO(X, x)$ such that $x \in U_0 \subset b\,\mathrm{cl}(U_0) \subset U$. Therefore, we obtain $f(b\,\mathrm{cl}(U_0)) \subset b\,\mathrm{cl}(V)$. This shows that f is θ-b-irresolute. □

Lemma 3.7. [8] *For the subsets A and B of X, $b\,\mathrm{cl}(A \times B) \subset b\,\mathrm{cl}(A) \times b\,\mathrm{cl}(B)$.*

Theorem 3.8. *A function $f : X \to Y$ is θ-b-irresolute if the graph function $g : X \to X \times Y$ of f, defined by $g(x) = (x, f(x))$ for each $x \in X$, is θ-b-irresolute.*

Proof. Suppose that g is θ-b-irresolute. Let $x \in X$ and $V \in BO(Y, f(x))$. Then $X \times V$ is a b-open set of $X \times Y$ containing $g(x)$. Since g is θ-b-irresolute, there exists $U \in BO(X, x)$ such that $g(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(X \times V)$. By Lemma

3.7, $b\,\mathrm{cl}(X \times V) \subset (X \times b\,\mathrm{cl}(V))$. Therefore, we obtain $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$. This shows that f is θ-b-irresolute. □

Lemma 3.9. [1, 5] *Let A and X_0 be subsets of a space X.*

(i) *If $A \in BO(X)$ and $X_0 \in \alpha O(X)$, then $A \cap X_0 \in BO(X_0)$;*
(ii) *If $A \in BO(X_0)$ and $X_0 \in \alpha O(X)$, then $A \in BO(X)$.*

Lemma 3.10. [8] *Let A and X_0 be subsets of a space X such that $A \subset X_0 \subset X$. Let $b\,\mathrm{cl}_{X_0}(A)$ denote the b-closure of A with respect to the subspace X_0.*

(i) *If X_0 is α-open in X, then $b\,\mathrm{cl}_{X_0}(A) \subset b\,\mathrm{cl}(A)$;*
(ii) *If $A \in BO(X_0)$ and $X_0 \in \alpha O(X)$, then $b\,\mathrm{cl}(A) \subset b\,\mathrm{cl}_{X_0}(A)$.*

Theorem 3.11. *If $f : X \to Y$ is θ-b-irresolute and X_0 is an α-open subset of X, then the restriction $f_{|X_0}\colon X_0 \to Y$ is θ-b-irresolute.*

Proof. For any $x \in X_0$ and any $V \in BO(Y, f(x))$, there exists $U \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$ since f is θ-b-irresolute. Let $U_0 = U \cap X_0$, then by Lemmas 3.9 and 3.10, $U_0 \in BO(X_0, x)$ and $b\,\mathrm{cl}_{X_0}(U_0) \subset b\,\mathrm{cl}(U_0)$. Therefore, we obtain $(f_{|x_0})\ (b\,\mathrm{cl}_{X_0}(U_0)) = f(b\,\mathrm{cl}_{X_0}(U_0)) \subset f(b\,\mathrm{cl}(U_0)) \subset f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$. This shows that $f_{|X_0}$ is θ-b-irresolute. □

Theorem 3.12. *A function $f : X \to Y$ is θ-b-irresolute if for each $x \in X$ there exists $X_0 \in \alpha O(X, x)$ such that the restriction $f_{|X_0}\colon X_0 \to Y$ is θ-b-irresolute.*

Proof. Let $x \in X$ and $V \in BO(Y, f(x))$. There exists $X_0 \in \alpha O(X, x)$ such that $f_{|X_0}\colon X_0 \to Y$ is θ-b-irresolute. Thus, there exists $U \in BO(X_0, x)$ such that $(f_{|X_0})\ (b\,\mathrm{cl}_{X_0}(U)) \subset b\,\mathrm{cl}(V)$. By Lemmas 3.9 and 3.10, $U \in BO(X, x)$ and $b\,\mathrm{cl}(U) \subset b\,\mathrm{cl}_{X_0}(U)$. Hence, we have $f(b\,\mathrm{cl}(U)) = (f_{|X_0})\ (b\,\mathrm{cl}(U)) \subset (f_{|X_0})\ (b\,\mathrm{cl}_{X_0}(U)) \subset b\,\mathrm{cl}(V)$. This shows that f is θ-b-irresolute. □

Corollary 3.13. *Let $\{U_\alpha : \alpha \in \wedge\}$ be an α-open cover of a topological space X. A function $f : X \to Y$ is θ-b-irresolute if and only if the restriction $f_{|U_\alpha}\colon U_\alpha \to Y$ is θ-b-irresolute for each $\alpha \in \wedge$.*

Proof. The proof follows from Theorems 3.11 and 3.12. □

Theorem 3.14. *Let $f : X \to Y$, $g : Y \to Z$ be functions and $g \circ f\colon X \to Z$ be the composition. Then the following properties hold:*

(i) *If f and g are b-θ-irresolute, then $g \circ f$ is θ-b-irresolute;*
(ii) *If f is strongly b-irresolute and g is weakly b-irresolute, then $g \circ f$ is θ-b-irresolute;*
(iii) *If f is weakly b-irresolute and g is θ-b-irresolute, then $g \circ f$ is weakly b-irresolute;*

(iv) *If f is θ-b-irresolute and g is strongly b-irresolute, then $g \circ f$ is strongly b-irresolute.*

Proof. The proof follows from the definitions. □

4. Graphs of θ-b-irresolute functions

Definition 4.1. *A topological space X is said to be b-T_2 [8] if for each pair of distinct points x and y in X, there exists $U \in BO(X, x)$ and $V \in BO(X, y)$ such that $b\,\mathrm{cl}(U) \cap b\,\mathrm{cl}(V) = \varnothing$.*

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x))\colon x \in X\}$ of $X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 4.2. *The graph $G(f)$ of a function $f : X \to Y$ is said to be strongly b-closed [8] (resp. b-θ-closed [8]) if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in BO(X, x)$ and $V \in BO(Y, y)$ such that $(b\,\mathrm{cl}(U) \times V) \cap G(F) = \varnothing$ (resp. $(b\,\mathrm{cl}(U) \times b\,\mathrm{cl}(V)) \cap G(f) = \varnothing$).*

Lemma 4.3. *The graph $G(f)$ of a function $f : X \to Y$ is b-θ-closed in $X \times Y$ if and only if for each point $(x, y) \in (X \times Y) - G(f)$, there exist $U \in BO(X, x)$ and $V \in BO(Y, y)$ such that $f(b\,\mathrm{cl}(U)) \cap b\,\mathrm{cl}(V) = \varnothing$.*

Proof. The proof follows from the definitions. □

Theorem 4.4. *If $f : X \to Y$ is θ-b-irresolute and Y is b-T_2, then $G(f)$ is b-θ-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. It follows that $f(x) \neq y$. Since Y is b-T_2, there exist b-open set V and W in Y containing $f(x)$ and y, respectively, such that $b\,\mathrm{cl}(V) \cap b\,\mathrm{cl}(W) = \varnothing$. Since f is θ-b-irresolute, there exists $U \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U)) \subset b\,\mathrm{cl}(V)$. Therefore, $f(b\,\mathrm{cl}(U)) \cap b\,\mathrm{cl}(W) = \varnothing$ and by Lemma 4.3, $G(f)$ is b-θ-closed in $X \times Y$. □

Recall that a space X is said to be b-T_2 [8] if for any pair of distinct points x, y of X, there exist disjoint b-open sets U and V such that $x \in U$ and $y \in V$.

Theorem 4.5. *If $f : X \to Y$ is strongly b-irresolute and Y is b-T_2, then $G(f)$ is b-θ-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. It follows that $f(x) \neq y$. Since Y is b-T_2, there exist b-open sets V and W in Y containing $f(x)$ and y, respectively, such that $V \cap W = \varnothing$ and hence $V \cap b\,\mathrm{cl}(W) = \varnothing$. Since f is strongly b-irresolute, there exists $U \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U)) \subset V$. Therefore, $f(b\,\mathrm{cl}(U)) \cap b\,\mathrm{cl}(W) = \varnothing$ and by Lemma 4.3, $G(f)$ is b-θ-closed in $X \times Y$. □

Theorem 4.6. *Let $f, g : X \to Y$ be functions. If $G(f)$ is b-θ-closed and g is θ-b-irresolute, then the set $\{(x_1, x_2){:}f(x_1) = g(x_2)\}$ is b-θ-closed in the product space $X \times X$.*

Proof. Let $A = \{(x_1, x_2) : f(x_1) = g(x_2)\}$. Suppose $(x_1, x_2) \notin A$. Then $f(x_1) \neq g(x_2)$ and hence $(x_1, g(x_2)) \notin G(f)$. Since $G(f)$ is b-θ-closed, there exist $U \in BO(X, x_1)$ and $W \in BO(Y, g(x_2))$ such that $f(b\,\mathrm{cl}(U)) \cap b\,\mathrm{cl}(W) = \varnothing$. Since g is θ-b-irresolute, there exists $U_0 \in BO(X, x_2)$ such that $g(b\,\mathrm{cl}(U_0)) \subset b\,\mathrm{cl}(W)$ and hence $f(b\,\mathrm{cl}(U)) \cap g(b\,\mathrm{cl}(U_0)) = \varnothing$. Therefore, we obtain $(b\,\mathrm{cl}(U) \times b\,\mathrm{cl}(U_0)) \cap A = \varnothing$ and hence A is b-θ-closed. □

Theorem 4.7. *If $f : X \to Y$ is a θ-b-irresolute function and Y is b-T_2, then the subset $A = \{(x, y){:}\ f(x) = f(y)\}$ is b-θ-closed in $X \times X$.*

Proof. Since f is θ-b-irresolute and Y is b-T_2, by Theorem 4.4, $G(f)$ is b-θ-closed. Therefore, by Theorem 4.6, A is b-θ-closed. □

Definition 4.8. *A topological space X is said to be*

(i) *b-closed* [8] *if every cover of X by b-open sets has a finite subcover whose b-closures cover X;*
(ii) *countably b-closed* [8] *if every countable cover of X by b-open sets has a finite subcover whose b-closures cover X.*

A subset K of a space X is said to be b-closed relative to X [8] if for every cover $\{V_\alpha{:}\ \alpha \in \wedge\}$ of K by b-open sets of X, there exists a finite subset $\wedge_0$ of $\wedge$ such that $K \subset \bigcup \{b\,\mathrm{cl}(V_\alpha){:}\ \alpha \in \wedge_0\}$.

Theorem 4.9. *If $f : X \to Y$ is θ-b-irresolute function and K is b-closed relative to X, then $f(K)$ is b-closed relative to Y.*

Proof. Suppose that $f : X \to Y$ is θ-b-irresolute and K is b-closed relative to X. Let $\{V_\alpha{:}\ \alpha \in \wedge\}$ be a cover of $f(K)$ by b-open sets of X. For each point $x \in K$, there exists $\alpha(x) \in \wedge$ such that $f(x) \in V_{\alpha(x)}$. Since f is θ-b-irresolute, there exists $U_x \in BO(X, x)$ such that $f(b\,\mathrm{cl}(U_x)) \subset b\,\mathrm{cl}(V_{\alpha(x)})$. The family $\{U_x{:}\ x \in K\}$ is a cover of K by b-open sets of X and hence there exists a finite subset K_1 of K such that $K \subset \bigcup_{x \in K_1} b\,\mathrm{cl}(U_x)$. Therefore, we obtain $f(K) \subset \bigcup_{x \in K_1} b\,\mathrm{cl}(V_{\alpha(x)})$. This shows that $f(K)$ is b-closed relative to Y. □

Corollary 4.10. *If $f : X \to Y$ be a θ-b-irresolute surjection. Then the following properties hold:*

(i) *If X is b-closed, then Y is b-closed;*
(ii) *If X is countably b-closed, then Y is countably b-closed.*

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