SARAJEVO JOURNAL OF MATHEMATICS
Vol.2 (15) (2006), 137–146

THE SEMANTIC DISTANCE, FUZZY DEPENDENCY AND FUZZY FORMULAS

NEDŽAD DUKIĆ

Abstract. In this paper we establish a connection between one fragment fuzzy logic and the theory of fuzzy functional dependencies on the basic of the semantic distance. We give a way to interpret fuzzy functional dependencies as formulas in fuzzy logic. For a set of fuzzy dependencies F and single fuzzy functional dependency f, we show that F implies f as fuzzy functional dependencies if and only if F implies f under the logic interpretation.

1. Introduction

In real word applications, data are often partially known or ambiguous. The classical relational model does not deal with this information. According to the classic relation database [2, 7] all the information in it, have to involve precisely defined values (atomic).

In an extension, a variety of null values have been introduced to model unknown or not-applicable data values [6].

The other way of considering this imprecise information is the involving of fuzzy value to the domain of an attribute [1]. Imprecise information has been studied in Zadeh's fuzzy set theory and fuzzy logic [8]. Fuzzy set theory and fuzzy logic provide a mathematical framework to deal with the imprecise information in fuzzy relational databases.

Approaches to the representation of inexact information in relation database theory include fuzzy membership values, similarity relationships and possibility distributions.

In a fuzzy set each element of the set has an associated degree of membership. The degree of membership is a real number between zero and one and measures the extent to which an element is in a fuzzy set [4, 5].

2000 *Mathematics Subject Classification.* 03B52.

Key words and phrases. Fuzzy relation database, fuzzy functional dependency, fuzzy logic.

As an extension of the degree of membership concept for sets of elements, we have a similarity relationship. Here the domain elements are considered as having varying degrees of similarity, replacing the idea of exact equality and inequality.

To deal with a fuzzy data constraint, Zadeh has introduced the concept of particularization (restriction) of a fuzzy relation due to a fuzzy proposition. The formed formulas of the first order calculus can be used to represent integrity constraints in a classical relational databases, fuzzy integrity constraint can be represented by suitable fuzzy propositions. The particularization of a fuzzy relational database due to a set of fuzzy integrity constraints can be computed by combining the fuzzy propositions associated with these integrity constraints according to the rules of fuzzy calculus.

Our primary aim in this paper is to establish a connection between the theory of fuzzy functional dependency on the basis of the semantic distance and one fragment of fuzzy logic. So it will be shown that if relation r satisfies a fuzzy functional dependence then its truth value of the belonging fuzzy formula is greater than 0.5 and vice verse.

Therefore, we will establish the equivalence of the calculation of one part of fuzzy logic and fuzzy functional dependence. After establishing this equivalence, it is possible to apply the rules of deduction in fuzzy logic on the calculus of fuzzy functional dependency on the basis of the semantic distance.

2. Fuzzy semantic distance

A fuzzy instance r on the relation scheme $R\{A_1,\ldots,A_n\}$ is subset of the Cartesian product of $dom(A_1)\,x\ldots x\,dom(A_n)$.

There are various descriptions of fuzzy values [8]. For example:

1. The Zadeh description. Let $dom(A_i)$ be a universe of discourse. A fuzzy subset X_i in $dom(A_i)$ is characterized by a member function $f_i : X_i \to [0,1]$ where $f_i(u)$ for each $u \in$dom(A$_i$) denotes the grade of membership of u in the fuzzy subset X_i.

The Zadeh description of the semantic distance $SD(f_1, f_2)$ between fuzzy attribute values: Let $f_1(X)$ and $f_2(X)$ be two fuzzy values. The semantic distance from $f_1(X)$ to $f_2(X)$ is defined by some norm $\|f_1(X)\ - f_2(X)\|$. For example $SD(f_1, f_2) = \max_{x\in dom(A_i)} |f_1(X) - f_2(X)|$.

2. The center number description. A fuzzy subset X_i in $dom(A_i)$ is characterized by $(c, r)/p$. It expresses that this fuzzy subset lies in the spherical region, where c is the center of the sphere, r is the radius of the sphere and $0\leq p \leq 1$.

The center number description of the semantic distance. Let $f_1 = (c_1, r_1)/p_1$ and $f_2 = (c_2, r_2)/p_2$ be two fuzzy values.

$$SD(f_1, f_2) = u_1 * g(c_1, c_2q) + u_2 * |r_1 - r_2| + u_3 * |p_1 - p_2|\,,$$

where u_1, u_2, u_3 are the weight coefficients and $u_1, u_2, u_3 \geq 0$, $u_1+u_2+u_3 = 1$.

3. Fuzzy functional dependency

3.1. **Fuzzy semantic distance.**

In a classic relation database functional dependency [2, 7] is a statement that describes a semantic constraint on data.

Let r be any relation instance on the scheme $R\{A_1, \ldots, A_n\}$, U be the universal set of attributes $A_{1,} \ldots, A_n$ and both X and Y be subsets of U. Relation instance r is said to satisfy the functional dependency $X \to Y$ if, for ever pair of tuples t_1 and t_2 in r, $t_1[X] = t_2[X]$ implies $t_1[Y] = t_2[Y]$.

But the definition of functional dependency is not directly applicable to fuzzy relational databases because it is based on the concept of equality.

There are several way to correct the definition of fuzzy functional dependency [3, 6, 9].

Two tuples are duplicates in a classical relation instance if each attribute value in the tuples are equivalent.

Definition 3.1.1. *The complement of $SD(f_1, f_2)$ is called the semantic similarity $SS(f_1, f_2)$.*

That is $SS(f_1, f_2) = 1 - SD(f_1, f_2)$ is the fuzzy similarity relation over a universe of discourse U i.e.

$SS(u, u) = 1$ *(reflexivity),*

$SS(u, v) = SS(v, u)$ *(symmetry).*

Definition 3.1.2. *Let r be a fuzzy instance $t_i(A_1, \ldots, A_n)$ and $t_j(A_1, \ldots, A_n)$ $(t_i, t_j \in r, i \neq j)$ duplicates. If*

i) $SD(t_i[A_k], t_j[A_k]) \leq \theta$ *for $k = 1, \ldots, n$ and $\theta \in [0, 1]$ is a given priority.*

ii) *The duplicate of the attribute set X for any two tuples t_i and t_j present in the relation instance r is given as*

$$SS(t_i[X], t_j[X]) = \min_{A_k \in X} \{SS(t_i[A_k], t_j[A_k])\}\,.$$

3.2. **Properties of the duplicate of attribute.**

Proposition 3.2.1. *If $X \supseteq Y$, then $SS(t_i[Y], t_j[Y]) \geq SS(t_i[X], t_j[X])$ for any t_i and t_j in r.*

Proof. Let $Y = \{A_1, \ldots, A_n\}, n \geq 0$ and $X = \{A_1, \ldots, A_{n,} A_{n+1}, \ldots, A_m\}$ $m \geq n$. Then for any two tuples in r,

$$SS(X - t_i[Y], t_j[Y]) = \min(SS(t_i[A_{n+1}], t_j[A_{n+1}], \ldots, SS(t_i[A_m], t_j[A_m]))$$

and

$$SS(t_i[X], t_j[X]) = \min(SS(t_i[A_1], t_j[A_1], \ldots, SS(t_i[A_m], t_j[A_m])).$$

$SS(t_i[X], t_j[X])$ can be rewriten as

$$SS(t_i[X], t_j[X]) = \min(SS(t_i[Y], t_j[Y]), SS(X - t_i[Y], t_j[Y]))$$

which implies

$$SS(t_i[Y], t_j[Y]) \geq SS(t_i[X], t_j[X]).$$

□

Proposition 3.2.2. *If $X = \{A_1, \ldots, A_n\}$ and $SS(t_i[A_k], t_j[A_k]) \geq \theta$, for all $k, 1 \leq k \leq n$, then $SS(t_i[X], t_j[X]) \geq \theta$ for any t_i and t_j in r.*

Proof. Let $X = \{A_1, \ldots, A_n\}$ and $SS(t_i[A_k], t_j[A_k]) \geq \theta$ for all $k, 1 \leq k \leq n$, for two tuples t_i and t_j in r. Then

$$SS(t_i[X], t_j[X]) \geq \theta$$

follows from Definition 3.0.2. ii). □

3.3. **Fuzzy functional dependencies.**

Definition 3.3.1. *Let r be any fuzzy relation instance on scheme $R(A_1, \ldots, A_n)$, U be the universal set of attributes $A_1, \ldots, A_n$, and both X and Y be subsets of U. A fuzzy relation instance r is said to satisfy the fuzzy functional dependency $(FFD) X \rightarrow Y$ if, for every pair of tuples t_1 and t_2 in r,*

$$SS(t_i[X], t_j[X]) \leq SS(t_i[Y], t_j[Y]).$$

3.4. **Inference rules for fuzzy functional dependency.**

IR1 *Reflexive* rule for fuzzy functional dependency:

$$if X \supseteq Y \text{ then } X \rightarrow Y \text{ holds.}$$

IR2 *Augmentation* rule for fuzzy functional dependency:

$$\{X \rightarrow Y\} \Rightarrow XZ \rightarrow YZ.$$

IR3 *Transitivity* rule for fuzzy functional dependency:

$$\{X \rightarrow Y, Y \rightarrow Z\} \Rightarrow X \rightarrow Z.$$

4. Fuzzy logic and resolution principle

Fuzzy logic is based on the concepts of fuzzy sets and symbolic logic. Logic operators of conjunction, disjunction and negation are defined as follows,

a) $x_1 \wedge x_2 = \min(x_1, x_2)$
b) $x_1 \vee x_2 = \max(x_1, x_2)$
c) $\neg x = 1 - x$

where $x_i (i = 1, 2, \dots, n)$ variable in $[0, 1]$, see [4, 5, 8].

In fuzzy logic, the truth value of a formula can assume any value in the interval $[0, 1]$ and is used to indicate the degree of truth represented by the formula.

4.1. Satisfiability in fuzzy logic.

Definition 4.1.1. *A formula $f \in S$, where is S set of a fuzzy formulas is said to satisfy in interpretation I, if truth value of a formula $T(f) \geq 0.5$ under I. An interpretation I is said to be false S if $T(f) \leq 0.5$.*

A formula is said to be unsatisfiable if it is false for every interpretation of it.

5. Fuzzy functional dependency on the basis of the semantic distance and fuzzy formulas

In this section we establish a connection between fuzzy logic and the theory of fuzzy functional dependencies on the basis of semantic distance. We give a way to interpret fuzzy functional dependencies as formulas in fuzzy logic. For a set of fuzzy dependencies F and single fuzzy functional dependency f, we show that F implies f as fuzzy functional dependencies if and only if F implies f under the logic interpretation.

The correspondence between fuzzy functional dependencies and fuzzy formulas is direct. Let $X \to Y$ be a fuzzy functional dependency where $X = A_1 A_2 \dots A_m$ and $Y = B_1 B_2 \dots B_n$. The corresponding logical formula is

$$(A_1 \wedge A_2 \wedge \dots \wedge A_m) \to (B_1 \wedge B_2 \wedge \dots \wedge B_n).$$

Let r be a fuzzy relation over schema R with exactly two tuples. A fuzzy relation r can be used to define a truth assignment, for attributes in R when they are considered as fuzzy variables.

Definition 5.0.2. *Let $R = \{A_1, A_2, \dots, A_m\}$ be a relation schema and let $r = \{t_1, t_2\}$ be a two tuple relation on R. The truth assignment for r, denoted i_r, is the function from R to $[0, 1]$ defined by*

$$i_r(A_k) \begin{cases} > [0.5, 1] & \textit{if } SS(t_i[A_k], t_j[A_k]) \geq \theta \in [0, 1], \\ \leq [0, 0.5] & \textit{if } SS(t_i[A_k], t_j[A_k]) < \theta. \end{cases}$$

The following theorem enables equivalence between fuzzy functional dependence and fuzzy formulas. Therefore by this theorem the mentioned equivalence will be proved taking for the fuzzy formulas the following:

$$X \Rightarrow Y = \max(1 - X, Y) \quad \text{(Kleen-Dienes)}.$$

Theorem 5.1. *Let $X \rightarrow Y$ be a FFD over relation scheme R and let r be a relation on R with two tuples. An FFD $X \rightarrow Y$ is satisfied by relation r if and only if $X \Rightarrow Y$ is satisfied under the truth assignments i_r.*

Proof. Let us assume, as first, that relation r satisfies FFD $X \rightarrow Y$ i.e. suppose

$$SS(t_i[X], t_j[X]) \leq SS(t_i[Y], t_j[Y])$$

where is $X = \{A_1, A_2, \ldots, A_m\}$ and $Y = \{B_1, B_2, \ldots, B_n\}$.

Let us assume to the contrary that the assertion that assignments

$$F : (A_1 \wedge A_2 \wedge, \ldots, \wedge A_m) \rightarrow (B_1 \wedge B_2 \wedge, \ldots, \wedge B_n)$$

is false under interpretation $i_{r'}$.

Then it follows that in interpretation $i_{r'}$ that $i_{r'}(F) \leq 0.5$, and respectively $i_{r'}(F) = i_{r'}((A_1 \wedge A_2 \wedge, \ldots, \wedge A_m) \rightarrow (B_1 \wedge B_2 \wedge, \ldots, \wedge B_n)) = \max(\min(1 - i_{r'}(A_1), 1 - i_{r'}(A_2), \ldots, 1 - i_{r'}(A_m)), \min(i_{r'}(B_1), i_{r'}(B_2), \ldots, i_{r'}(B_n)) \leq 0.5$. Then we have

$$i_{r'}(F) = \begin{cases} i'_r(A_i) > 0.5, & \forall i = 1, 2, \ldots, m, \\ i'_r(B_j) \leq 0.5 & \exists j = 1, 2, \ldots, n. \end{cases}$$

If $i'_r(A_i) > 0.5$, $\forall i = 1, 2, \ldots, m$ is valid then according to Definition 5.0.2 $SS(t_1[A_i], t_2[A_i]) \geq \theta$.

Based on the Definition 3.1.2. ii) we have

$$SS(t_i[X], t_j[X]) = \min_{A_k \in X} \{SS(t_i[A_k], t_j[A_k])\}.$$

Now, on the basis of Proposition 3.2.2 we also have $SS(t_1[X], t_2[X]) \geq \theta$. Because of the assumption that FFD is satisfied, we have

$$\begin{aligned} SS(t_1[Y], t_2[Y]) &= \min(SS(t_1[B_1], t_2[B_1], \ldots, SS(t_1[B_n], t_2[B_n]) \\ &\geq (SS(t_1[X], t_2[X])) \\ &= \min(SS(t_1[A_1], t_2[A_1], \ldots, SS(t_1[A_m], t_2[A_m])) \\ &\geq \theta. \end{aligned}$$

This implies that $SS(t_1[B_j], t_2[B_j]) \geq \theta$ for each $j = 1, 2, \ldots, n$. So it follows that $i'_r(B_j) > 0.5$, which is contrary to $i'_r(B_j) \leq 0.5$.

Therefore the assertion is valid if the relation r satisfies FFD $X \rightarrow Y$, then its assignment fuzzy formula is satisfied in the interpretation $i_{r'}$.

Now we prove the converse of theorem. Assume that F satisfies in interpretation $i_{r'}$. Then $i'_r(F) = \max(\min(1 - i'_r(A_1), 1 - i'_r(A_2), \dots, 1 - i'_r(A_m)), \min(i'_r(B_1), i'_r(B_2), \dots, i'_r(B_n)) > 0.5$. which results in

i) $i_{r'}(A_1 \wedge A_2 \wedge, \dots, \wedge A_m) \leq 0.5$ or

ii) $i_{r'}(B_1 \wedge B_2 \wedge, \dots, \wedge B_n) > 0.5$.

Suppose i) is valid, then $i_{r'}(A_1 \wedge A_2 \wedge, \dots, \wedge A_m) = \min(i_{r'}(A_1), i_{r'}(A_2), \dots, i_{r'}(A_m))$; hence $i_{r'}(A_j) \leq 0.5$ for some j in $\{1, 2, \dots, m\}$, from which it follows that $SS(t_1[A_j], t_2[A_j]) < \theta$ for some j in $\{1, 2, \dots, m\}$. Then $SS(t_1[X], t_2[X]) < \theta$.

From this it follows that the relation satisfies FFD $X \to Y$.

Suppose ii) is valid i.e. $i_{r'}(B_1 \wedge B_2 \wedge, \dots, \wedge B_n) > 0.5$ for each $i = 1, 2, \dots, n$. Then $\min(i_{r'}(B_1), i_{r'}(B_2), \dots, i_{r'}(B_n)) > 0.5$ and $i_{r'}(B_i) > 0.5$ for each $i = 1, 2, \dots, n$ from which it follows that $SS(t_1[B_i], t_2[B_i]) \geq \theta$.

Hence it follows that r satisfies FFD $X \to Y$. □

In the following theorem we are going to show that if the relation r satisfies a set of fuzzy functional dependencies F and does not satisfy the dependency $X \to Y$ then there exists two tuples sub-relations of the relation r, which satisfies all the fuzzy functional dependencies from set F and does not satisfy the dependency $X \to Y$.

Theorem 5.2. *Let $X \to Y$ be an FFD over the scheme R, and $\{A_1, A_2, \dots, A_m\} = X \subseteq R$, and $\{B_1, B_2, \dots, B_n\} \subseteq R$, and let F be a set of FFDs over R. Then the following is true:*

1) *$F \Rightarrow X \to Y$ if and only if*

2) *$F \Rightarrow X \to Y$ in the world of two tuple relations.*

Proof. Obviously 1) implies 2). Now we show that 2) implies 1).

Let us assume that 2) does not imply 1). In that case there is some relation r satisfied by all the fuzzy functional dependencies from F, that does not satisfy the dependency $X \to Y$. This means that there exists elements t_1 and t_2 in r, for which

$$SS(t_i[X], t_j[X]) > SS(t_i[Y], t_j[Y]).$$

□

Let be $r^* = \{t_1, t_2\}$. It is obvious that r^* satisfies all the FFDs from F, but does not satisfy the dependency $X \to Y$. This is shown by following.

Lemma 5.1. *Let r be a relation, let F be a set of FFDs on R, and let $X \to Y$ be a single FFD on R. If relation r satisfies all the FFDs from the set F and violates fuzzy dependency $X \to Y$, then some two tuple sub-relation r^* of r satisfies F and violates $X \to Y$.*

This is a contradiction of our assumption.

Theorem 5.3. *Let $X \to Y$ be an FFD over relation scheme R and let F be a set of FFDs over R. Then F implies $X \to Y$ in the world of two tuple relations, if and only if F implies $X \Rightarrow Y$ when FFDs are interpreted as fuzzy formulas.*

Proof. Assume that $i_r : R \to [0,1]$ is an interpretation where every formula is satisfied, which is generated by FFDs from set F, and let a formula which is generated by the dependency $X \to Y$ be false. Let us consider $Z = \{A \in R : i_r(A) > 0.5\}$.

Let r_z be a fuzzy relation instance with two tuples t_1 and t_2 as shown in Fig. 1. We choose the set $\{a, b\}$ as the domain of each attributes in R, where $a = a_1, \dots, a_p$, and $b = b_1, \dots, b_q$, $(p \geq 1, q \geq 1)$. Let $SS(t_1[A], t_2[A]) \geq \theta$ for any attribute set A in r_z,

	Attributes of Z	other attributes
t_1	$a, \dots, a$	$a, \dots, a$
t_2	$a, \dots, a$	$b, \dots, b$

Fig. 1. The fuzzy relation instance r$_z$.

Namely $r_z = \{t_1, t_2\}$ where $t_1 = a, \dots,$ a for each attribute A from R, and let t_2 be defined as

$$t_2 = \begin{cases} a, \dots, a, & R \in Z \\ b, \dots, b, & R \notin Z \end{cases}.$$

We will prove that relation r_z defined in such way satisfies each fuzzy functional dependency from F. To be able to prove this, let $U \to V$ be any fuzzy functional dependency from F for which

$$SS(t_1[U], t_2[U]) \geq \theta.$$

Due to the definition t_1 and $t_2 = a, \dots, a$ for each attribute A from U, we have $SS(t_1[A], t_2[A]) \geq \theta$. This means that $i_r(A) > 0.5$ for each A from U. There fore $U \subseteq Z$, i.e.

$$i_r(U) > 0.5. \qquad (^*)$$

If $SS(t_1[V], t_2[V]) \geq \theta$ does not hold, $t_1 = a, \dots, a$ and $t_2 = b, \dots, b$ for some attribute A from V, namely $SS(t_1[A], t_2[A]) < \theta$. From this we have that A does not belong set Z, and therefore $i_r(A) < 0.5$, and also $i_r(V) < 0.5$.

Based on this and (*) we have by the Kleens-Diens implication

$$i_r(U \to V) = \max(i_r(1 - U),\ i_r(V)) \leq 0.5$$

which is a contradiction of our first assumption.

Now we prove that r_z does not satisfy the fuzzy functional dependency $X \to Y$ i.e.

$$SS(t_i[X], t_j[X]) > SS(t_i[Y], t_j[Y]).$$

By assumption the fuzzy formula is false in the interpretation i_r and hence $i_r(X) > 0.5$ and

$$i_r(Y) \leq 0.5. \qquad (^{**})$$

Let assume that

$$SS(t_1[X], t_2[X]) \geq \theta.$$

If $SS(t_1[Y], t_2[Y]) \geq \theta$ holds, then $Y \subseteq Z$, namely $i_r(B_j) > 0.5$ for each $j = 1, 2, \ldots, n$, $B_j \in Y$. This implies that $i_r(Y) > 0.5$, which is a contradiction of (**).

Now we prove the converse. Assume the converse is false, i.e. that it does not hold that from the set of FFDs F follows FFD $X \to Y$. Then here exist two tuples relation $r = \{t, t'\}$ which satisfy each FFDs from F, but does not satisfy FFD $X \to Y$.

By the above mentioned description the interpretation i_r is defined by the relation r, formulas $U_1 \wedge U_2 \wedge \cdots \wedge U_p \to V_1 \wedge V_2 \wedge \cdots \wedge V_q$, for $U \to V$ from F and formula

$$X_1 \wedge \cdots \wedge X_m \to Y_1 \wedge \cdots \wedge Y_n.$$

Let us now prove that the following holds

i) $i_r(U_1 \wedge \cdots \wedge U_p \to V_1 \wedge \cdots \wedge V_q)) > 0.5$ and
ii) $i_r((X_1 \wedge \cdots \wedge X_m) \to (Y_1 \wedge \cdots \wedge Y_n)) \leq 0.5$.

If i) is false then $i_r(U_i) > 0.5$ and $i_r(V_j) \leq 0.5$, namely

$$SS(t[P], t'[P]) \geq \theta$$

for each P from U and

$$SS(t[Q], t'[Q]) < \theta$$

for some Q from V.

This first inequality implies that $SS(t[U], t'[U]) \geq \theta$, and the second implies that $SS(t[V], t'[V]) \geq \theta$.

Therefore these taken together is a contradiction with starting assumption that r satisfies each fuzzy functional dependencies from F. Therefore i) is true.

Suppose ii) is false, then

iii) $i_r(X_i) \leq 0.5$ or
iv) $i_r(Y_j) > 0.5$.

If iii) holds, then $SS(t[A_i], t'[A_i]) < \theta$, for some $j = 1, 2, \ldots, m$, $A_i \in X$ and from these $SS(t[X], t'[X]) < \theta$. It is obvious that r satisfies the fuzzy functional dependency $X \to Y$, which is a contradiction of the opening assumption.

If iv) holds then $SS(t[B], t'[B]) \geq \theta$ for each $j = 1, 2, \ldots, n$, $B_j \in Y$ and from this $SS(t[Y], t'[Y]) \geq \theta$. Therefore we conclude that r satisfies the fuzzy functional dependency $X \to Y$, which is also a contradiction of our opening assumption. □

6. Conclusion

In this paper we proved the equivalence between the theory of the fuzzy functional dependencies on the basis of the semantic distance for fuzzy database and the fragment theory of fuzzy logic.

To achieve this aim, we introduced the definition of truth assignment of attributes in the relation r over the relation scheme R. Based on this, definition of fuzzy functional dependencies on the basis of semantic distance was attached to the fuzzy formula and we proved that if the relation r satisfies the fuzzy functional dependencies on the basis of semantic distance then the fuzzy formula is satisfied in the given interpretation and vice verse. The equivalence between the set of the fuzzy functional dependencies on the basis of semantic distance and the fuzzy formulas was proved as well.

References

[1] B. P. Buckles and F. E. Petry, *A fuzzy representation of data for relational database*, Fuzzy Sets Syst., 7 (3) (1982), 213–216.
[2] E. F. Codd, *A relational model of data for large shared databases*, ACM Comm., 13 (1970), 377–387.
[3] N. Dukić and Z. Avdagić, *Fuzzy functional dependency and the resolution principle*, Informatica, Vilnuis, 16 (1) (2005), 45–60.
[4] M. Mizumoto and H. J. Zimmermann, *Comparasion of fuzzy reasoning methods*, Fuzzy Set Syst., 8 (1982), 253–283.
[5] M. Mukaidono, *Fuzzy Deduction of Resolution Type*, in R. Yager (ed) Fuzzy Set and Possibility Theory (1986).
[6] K. V. S. V. N. Raju and A. K. Majumdar, *Fuzzy functional dependencies and lossless join decomposition of fuzzy relational database system*, ACM Trans. Database Syst., 13 (1988), 129–166.
[7] J. D. Ullman, *Principles of Database and Knowledge – Base System*, Computer Science Press, Rockville, 2^{nd}, (1982).
[8] L. A. Zadeh, *Calculus of Fuzzy Restriction*, in L. A. Zadeh et al., Eds, Fuzzy Sets and Their Applications to Cognitive and Decision Processes, Academic Press, NewYork, (1975).
[9] Wei-Yi Liu, *The fuzzy functional dependency on the basis of the semantic distance*, Fuzzy Sets Syst., 59 (1993), 173–179.

(Received: June 16, 2006)
(Revised: July 20, 2006)

Department of Mathematics
Faculty of Science
University of Sarajevo
Bosnia and Herzegovina
E–mail: ndukic@pmf.unsa.ba