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SEMI-SLANT SUBMANIFOLDS OF TRANS-SASAKIAN MANIFOLDS

VIQAR AZAM KHAN AND MERAJ ALI KHAN

Abstract. The purpose of the present paper is to study semi-slant submanifolds of a trans-Sasakian manifold.

1. Introduction

The study of slant immersions was initiated by B. Y. Chen [5]. A. Lotta [2] extended the notion to the setting of almost contact metric manifolds. A further modification in the subject, in terms of semi-slant submanifolds was introduced by N. Papaghiuc [12]. In fact, semi-slant submanifolds in almost Hermitian manifolds are defined on the lines of CR-submanifolds. In the setting of almost contact metric manifolds, semi-slant submanifolds are defined and investigated by J. L. Cabrerizo et. al [9]. These submanifolds are studied in further specialized settings of K-contact and Sasakian manifolds by J. L. Cabrerizo et. al [9]. They worked out the integrability conditions of the distributions involved on these submanifolds and studied the geometry of the leaves of these distributions. In the present note, our aim is to extend the study of the semi-slant submanifolds to the setting of trans-Sasakian manifolds.

So for as trans-Sasakian manifolds are concerned, in the Gray-Hervella classification of almost Hermitian manifolds [3], there appears a class W_4, of Hermitian manifolds which are closely related to a locally conformal Käehler manifolds. An almost contact metric structure on a manifold $\bar{M}$ is called a trans-Sasakian structure [10] if the product manifold $\bar{M} \times R$ belongs to class W_4. The class $C_6 \oplus C_5$ [11] coincides with the class of trans-Sasakian structure of type (α, β). We note that the trans-Sasakian structure of type $(0, 0)$ are cosymplectic [6], trans-Sasakian structure of type $(0, \beta)$ are $\beta-$ Kenmotsu and trans-Sasakian structure of type $(\alpha, 0)$ are $\alpha-$ Sasakian [7].

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In the present paper we study semi-slant submanifolds of a trans-Sasakian manifold. The paper is divided into four sections. In Section 2 we recall some necessary detail of a trans-Sasakian manifold. In Section 3, we have obtained integrability conditions of the distributions involved on a semi-slant submanifold of a trans-Sasakian manifold and consequently studied the geometry of the leaves of these distributions. In Section 4, some interesting results on a totally umbilical semi-slant submanifolds of trans-Sasakian manifolds have been obtained.

2. Preliminaries

Let $\bar{M}$ be an odd dimensional Riemannian manifold with a Riemannian metric g and Riemannian connection $\bar{\nabla}$. Denote by $T\bar{M}$ the Lie algebra of vector fields on $\bar{M}$. Then $\bar{M}$ is said to be an almost contact metric manifold [6], if there exist on $\bar{M}$ a tensor ϕ of type (1, 1), a vector field ξ, called structure vector field and η, the dual 1-form of ξ satisfying the following

$$(a)\ \ \phi^2 X = -X + \eta(X)\xi \qquad\qquad (b)\ \ g(X,\xi) = \eta(X) \tag{2.1}$$

$$\eta(\xi) = 1 \qquad\qquad \phi(\xi) = 0 \qquad\qquad \eta\circ\phi = 0 \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{2.3}$$

for any $X, Y \in T\bar{M}$. In this case

$$g(\phi X, Y) = -g(X, \phi Y). \tag{2.4}$$

Throughout, all the maps are assumed to be differentiable. The fundamental 2-form F on $\bar{M}$ is given by

$$F(X,Y) = g(X, \phi Y).$$

Moreover, on $\bar{M}$ if

$$(\bar{\nabla}_X \phi)Y = \alpha(g(X,Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X) \tag{2.5}$$

then $\bar{M}$ is said to be a trans-Sasakian manifold [10], where α, β are functions on $\bar{M}$. On a trans-Sasakian manifold $\bar{M}$, we have

$$\bar{\nabla}_X \xi = -\alpha(\phi X) + \beta(X - \eta(X)\xi). \tag{2.6}$$

Now, let M be a submanifold immersed in $\bar{M}$. The Riemannian metric induced on M is denoted by the same symbol g. Let TM and $T^{\perp}M$ be the Lie algebras of vector fields tangential to M and normal to M respectively and ∇ be the induced Levi-Civita connection on M, then the Gauss and Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{2.7}$$

$$\bar{\nabla}_X V = -A_V X + \nabla^{\perp}_X V \tag{2.8}$$

for any $X, Y \in TM$ and $V \in T^{\perp}M$. Where $\nabla^{\perp}$ is the connection on the normal bundle $T^{\perp}M$, h is the second fundamental form and A_V is the Weingarten map associated with V as

$$g(A_V X, Y) = g(h(X, Y), V). \tag{2.9}$$

For any $x \in M$ and $X \in T_xM$, we write

$$\phi X = TX + NX \tag{2.10}$$

where $TX \in T_xM$ and $NX \in T_x^{\perp}M$. Similarly, for $V \in T_x^{\perp}M$, we have

$$\phi V = tV + nV \tag{2.11}$$

where tV (resp. nV) is the tangential component (resp. normal component) of ϕV.

From (2.4) and (2.10), it is easy to observe that for each $x \in M$, and $X, Y \in T_xM$

$$g(TX, Y) = -g(X, TY) \tag{2.12}$$

and therefore $g(T^2X, Y) = g(X, T^2Y)$ which implies that the endomorphism $T^2 = Q$ is self adjoint. Moreover, it can be seen that the eigenvalues of Q belong to $[-1, 0]$ and that each non-vanishing eigenvalue of Q has even multiplicity. We define ∇T, ∇Q and ∇N by

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y \tag{2.13}$$

$$(\nabla_X Q)Y = \nabla_X QY - Q\nabla_X Y \tag{2.14}$$

$$(\nabla_X N)Y = \nabla_X^{\perp} NY - N\nabla_X Y \tag{2.15}$$

for any $X, Y \in TM$.

Throughout, the vector field ξ is assumed to be tangential to M, for otherwise M is simply anti-invariant (cf., [2]). For any $X, Y \in TM$ on using (2.6) and (2.7) we have the following

$$(a)\ \nabla_X \xi = -\alpha TX + \beta(X - \eta(X)\xi) \qquad (b)\ h(X, \xi) = -\alpha NX, \tag{2.16}$$

and by using (2.5), (2.7), (2.8), (2.9), (2.11), (2.13) and (2.15) we obtain

$$\begin{aligned}(\nabla_X T)Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)TX) \\ + A_{NY}X + th(X, Y)\end{aligned} \tag{2.17}$$

$$(\nabla_X N)Y = -\beta\eta(Y)NX - h(X, TY) + nh(X, Y) \tag{2.18}$$

3. Semi-slant subamnifold of trans-Sasakian manifolds

N. Papaghiuc in [12] introduced the notion of semi-slant submanifold of an almost Hermitian manifold. A semi-slant submanifold is a generalized version of CR-submanifold. Recently, Cabrerizo et.al. [9] gave the contact version of semi-slant submanifold and have found several interesting results in this setting. The purpose of this section is to study the semi-slant submanifolds in the setting of a trans-Sasakian manifold.

A submanifold M of an almost contact metric manifold $\bar{M}$ is said to be a slant submanifold if for any $x \in M$ and $X \in T_xM$, linearly independent to ξ, the angle between ϕX and T_xM is constant. The constant angle $\theta \in [0, \pi/2]$ is then called the slant angle of M in $\bar{M}$. If $\theta = 0$, the submanifold is invariant submanifold and if $\theta = \pi/2$ the submanifold is anti-invariant submanifold. If $\theta \neq 0$, $\pi/2$ then the submanifold is called a proper slant submanifold.

Let M be a submanifold of an almost contact metric manifold $\bar{M}$. Then M is said to be a semi-slant submanifold of $\bar{M}$ if there exist two orthogonal distributions D_1 and D_2 on M such that at each point $x \in M$

(i) T_xM admits the orthogonal direct decomposition i.e.,

$$T_xM = D_1 \oplus D_2 \oplus \langle \xi_x \rangle.$$

(ii) The distribution D_1 is an invariant distribution i.e., $\phi D_1 = D_1$

(iii) The distribution D_2 is slant with slant angle $\theta \neq 0$ and $\langle \xi \rangle$ denotes the distribution spanned by the structure vector field ξ.

For $\theta = \pi/2$, the semi-slant submanifold is semi-invariant submanifold. On a semi-slant subamnifold M for any $X \in TM$, we may write

$$X = P_1X + P_2X + \eta(X)\xi \tag{3.1}$$

where $P_1X \in D_1$ and $P_2X \in D_2$. Applying ϕ, (3.1) in view of (2.10) yields

$$\phi X = \phi P_1X + TP_2X + NP_2X. \tag{3.2}$$

Then it is easy to observe that

$$\phi P_1X = TP_1X, \quad NP_1X = 0 \quad \text{and} \quad TP_2X \in D_2.$$

Consequently, we find that

$$TX = \phi P_1X + TP_2X \quad \text{and} \quad NX = NP_2X. \tag{3.3}$$

Let μ denote the orthogonal complement of ϕD_2 in $T^{\perp}M$ i.e. $T^{\perp}M = \phi D_2 \oplus \mu$. Then it is easy to observe that μ is an invariant sub bundle of $T^{\perp}M$.

Now, we are in a position to work out the integrability conditions of the distributions D_1 and D_2 on a semi-slant submanifold of a trans-Sasakian manifold.

Lemma 3.1. *Let M be a semi-slant subamnifold of a trans-Sasakian manifold $\bar{M}$ then*

$$g([X,Y],\xi) = 2\alpha g(TX,Y) \tag{3.4}$$

for any $X, Y \in D_1 \oplus D_2$.

The assertion can be proved by using (2.16)(a) and (2.12). Consequently, we have

Corollary 3.1. *Let M be a semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$. Then the distributions D_1 and $D_1 \oplus D_2$ are not integrable on M in general.*

In particular on a $\beta-$Kenmotsu manifold, we have

Corollary 3.2. *The distribution $D_1 \oplus D_2$ on a semi-slant submanifold of a $\beta-$Kenmotsu manifold is integrable. Whereas D_1 is integrable if and only if*

$$h(X,\phi Y) = h(\phi X,Y)$$

for each $X, Y \in D_1$.

Proof. For any $X, Y \in D_1$ and $V \in T^{\perp}M$,

$$g(\bar{\nabla}_X \phi Y - \bar{\nabla}_Y \phi X, V) = g(h(X,\phi Y) - h(\phi X,Y), V)$$

which on using equation (2.5) and (3.2) yields

$$g(NP_2[X,Y],V) = g(h(X,\phi Y) - h(\phi X,Y),V). \tag{3.5}$$

The corollary follows in view of (3.4) and (3.5). □

Now, for the slant distribution, we have

Proposition 3.1. *Let M be a semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$. Then the slant distribution D_2 is integrable if and only if slant angle of D_2 is $\pi/2$ i.e. D_2 is anti-variant.*

Proof. For any $Z, W \in D_2$, by (3.4), we have

$$g([Z,W],\xi) = 2\alpha g(W,TZ).$$

If D_2 is integrable, then $T|_{D_2} = 0$ that means $\theta = \pi/2$.

Conversely, if sla$(D_2) = \pi/2$ then $\phi Z = NZ$ for each Z in D_2 and by equations (2.5), (2.7) and (2.8),

$$\phi\nabla_Z W + \phi h(Z,W) = -A_{NW}Z + \nabla^{\perp}_Z NW - \alpha g(Z,W)\xi$$

for each Z, W in D_2. Substituting W by Z in the above equation and subtracting the obtained relation from the same, we get

$$\phi[Z,W] = A_{NZ}W - A_{NW}Z + \nabla^{\perp}_Z NW - \nabla^{\perp}_W NZ \tag{3.6}$$

Further, by using equations (2.9), (2.4), (2.7), (2.5) and (2.8), it is easy to obtain that

$$A_{NZ}W = A_{NW}Z \tag{3.7}$$

for each Z, W in D_2. Now, equation (3.6) in view of equations (3.4), (2.1) and (3.7) gives

$$[Z, W] = \phi(\nabla^{\perp}_{Z} NW - \nabla^{\perp}_{W} NZ). \tag{3.8}$$

The right hand side of the above lies in D_2 because on using equation (2.5), (2.7) and (2.12), we see that

$$g(V, \nabla^{\perp}_{W} NZ) = -g(A_{\phi V}W, Z)$$

for all $V \in \mu$ and $Z, W \in D_2$. This shows that

$$g(\nabla^{\perp}_{Z} NW - \nabla^{\perp}_{W} NZ, V) = 0$$

i.e., $\nabla^{\perp}_{Z} NW - \nabla^{\perp}_{W} NZ$ lies in ND_2 for each Z, W in D_2, and thus from equation (3.8), $[Z, W] \in D_2$. □

Corollary 3.3. *Let M be a semi-slant submanifold of a $\beta-$ Kenmotsu manifold $\bar{M}$. Then the slant distribution D_2 is integrable if and only if*

$$P_1(\nabla_Z TW - A_{NW}Z - \nabla_W TZ + A_{NZ}W) = 0$$

for all $Z, W \in D_2$.

Proof. Making use of equations (2.5), (2.7) and (2.8) we have

$$g(T[Z, W], X) = g(\nabla_Z TW - A_{NW}Z - \nabla_W TZ + A_{NZ}W, X) \tag{3.9}$$

for any $Z, W \in D_2$ and $X \in D_1$. The corollary follows by equations (3.4) and (3.9). □

By equation (2.5), for any $Y \in TM$.

$$(\bar{\nabla}_\xi \phi)Y = 0$$

i.e.,

$$\bar{\nabla}_\xi \phi Y = \phi \bar{\nabla}_\xi Y. \tag{3.10}$$

In particular, for $X \in D_1$, equation (3.10) together with equations (2.7) and (2.16) implies that

$$\nabla_\xi X \in D_1. \tag{3.11}$$

The above observation together with (2.16)(a) yields

Lemma 3.2. *Let M be a semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$ then*

$$[X, \xi] \in D_1 \text{ and } [Z, \xi] \in D_2$$

for any $X \in D_1$ and $Z \in D_2$.

This leads to the following

Proposition 3.2. *Let M be a semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$ then*

(i) $D_1 \oplus \langle \xi \rangle$ *is integrable if and only if*

$$h(X, \phi Y) = h(Y, \phi X)$$

(ii) $D_2 \oplus \langle \xi \rangle$ *is integrable if and only if*

$$P_1(\nabla_Z TW - A_{NW}Z - \nabla_W TZ + A_{NZ}W) = 0$$

for each X, Y in D_1 and Z, W in D_2.

Proof. The statement (i) follows from equation (3.5) and Lemma 3.2. The statement (ii) follows from (3.9) and Lemma 3.2. □

The Nijenhuis tensor field with respect to T is given by

$$S(X, Y) = [TX, TY] + T^2[X, Y] - T[X, TY] - T[TX, Y]$$

for all $X, Y \in TM$. In particular, for $X \in D_1$ and $Z \in D_2$

$$S(X, Z) = (\bar{\nabla}_{TX}T)Z - (\bar{\nabla}_{TZ}T)X + T(\bar{\nabla}_Z T)X - T(\bar{\nabla}_X T)Z.$$

Using equation (2.17) the above equation becomes

$$S(X, Z) = A_{NZ}TX + th(TX, Z) - th(TZ, X) - TA_{NZ}X. \tag{3.12}$$

Lemma 3.3. *Let M be a semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$. If the distribution $D_1 \oplus \langle \xi \rangle$ is integrable and its leaves are totally geodesic in M, then*

(i) $h(D_1, D_1) \in \mu$.

(ii) $S(D_1, D_2) \in D_2$.

Proof. By hypothesis,

$$g(\nabla_X Y, Z) = 0$$

for any $X, Y \in D_1$ and $Z \in D_2$. That gives

$$g(\phi \bar{\nabla}_X Y, \phi Z) = 0$$

which on applying (2.5), (2.7) and (2.9), yields

$$g(h(X, \phi Y), NZ) = 0$$

This proves statement (i). To prove statement (*ii*) use (3.12) to get

$$g(S(X, Z), Y) = g(A_{NZ}TX + th(TX, Z) - th(TZ, X) - TA_{NZ}X, Y).$$

The right hand side of the above equation is zero in view of statement (i) and the Lemma is proved completely. □

Corollary 3.4. *If the invariant distribution D_1 on a semi-slant submanifold M of a $\beta-$Kenmotsu is integrable and its leaves are totally geodesic in M, then statement* (i) *and* (ii) *of Lemma* 3.3 *are true.*

Note. The above corollary is not true for the setting of semi-slant submanifold of a $\alpha-$Sasakian manifold as in this case D_1 is not integrable (cf., [1]) whereas D_1 is integrable on a semi-slant submanifold of a Kenmotsu manifold if and only if $h(X, \phi Y) = h(\phi X, Y)$ for each X, Y in D_1.

Lemma 3.4. *Let M be a semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$. If the slant distribution $D_2 \oplus \langle\xi\rangle$ is integrable and its leaves are totally geodesic in M, then*

(i) $h(D_1, D_2) \in \mu$.
(ii) $S(D_1, D_2) \in D_1$.

Proof. Suppose $D_2 \oplus \langle\xi\rangle$ is integrable and its leaves are totally geodesic, then for any $Z, W \in D_2$ and $X \in D_1$

$$g(\nabla_Z W, \phi X) = 0$$

which by applying (2.4), (2.5), (2.7) and (2.8) yields,

$$g(h(X, Z), NW) = 0,$$

that proves (i).

To prove (ii), on applying formula (3.12) and using the statement (i), we find that $g(S(X, Z), W) = 0$ for any $X \in D_1$ and $Z, W \in D_2$. This proves (ii) and the Lemma. □

Example. Let $R^{2n+1} = C^n \times R$ be the $(2n+1)-$ dimensional Euclidean space endowed with the almost contact metric structure (ϕ, ξ, η, g) defined by

$$\phi(x^1, x^2, \cdots, x^{2n}, t) = (-x^{n+1}, -x^{n+2}, \cdots, -x^{-2n}, x^1, x^2, \cdots, x^n, 0)$$

$$\xi = e^t \frac{\partial}{\partial t}, \quad \eta = e^t dt \quad \& \quad g = e^{2t} k.$$

where $(x^1, \cdots, x^{2n+1})$ are cartesian coordinates and k is the Euclidean Riemannian metric on R^{2n+1}. Then (ϕ, ξ, η, g) is a trans-Sasakian structure on R^{2n+1} which is not quasi-Sasakian. Moreover it is neither cosymplectic nor Sasakian $(cf., [10])$.

Let $1 < h < n$. Then the product $M_1 \times M_2 \times R$, where M_1 is a complex submanifold of C^h and M_2 is a slant submanifold of C^{n-h}, is a semi-slant submanifold of the trans-Sasakian manifold $(R^{2n+1}, \phi, \xi, \eta, g)$.

To be more precise, for $\theta \in [0, \pi/2]$

$$x(u_1, u_2, u_3, u_4, u_5) = (u_1, 0, u_3, 0, u_2, 0, u_4 \cos\theta, u_4 \sin\theta, u_5)$$

defines a 5-dimensional submanifold M in R^9 with the trans-Sasakian structure described above. Further,

$$e_1 = \frac{\partial}{\partial x^1} + x^5 \frac{\partial}{\partial t}; \;\; e_2 = \frac{\partial}{\partial x^5}, \;\; e_3 = \frac{\partial}{\partial x^3} + x^7 \frac{\partial}{\partial t};$$

$$e_4 = \cos\theta \frac{\partial}{\partial x^7} + \sin\theta \frac{\partial}{\partial x^8}; \;\; e_5 = \frac{\partial}{\partial t} = \xi$$

form an orthonormal frame of TM. If we define the distribution $D_1 = \langle e_1, e_2 \rangle$ and $D_2 = \langle e_3, e_4 \rangle$, then it is easy to check that the distribution D_1 is invariant under ϕ and D_2 is slant with slant angle θ. That is, M is a semi-slant submanifold of $(R^9, \phi, \xi, \eta, g)$ by definition.

4. Totally umbilical semi-slant submanifolds of trans-Sasakian manifolds

Definition. *A submanifold M is said to be totally umbilical submanifold if its second fundamental form satisfies*

$$h(X, Y) = g(X, Y)H \tag{4.1}$$

for all $X, Y \in TM$, where H is the mean curvature vector.

Before studying totally umbilical submanifold, we first establish the following preliminary result.

Proposition 4.1. *Let M be a semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$ with $h(X, TX) = 0$ for each $X \in D_1 \oplus \langle \xi \rangle$. If $D_1 \oplus \langle \xi \rangle$ is integrable then each of its leaves is totally geodesic in M as well as in $\bar{M}$.*

Proof. For any $X \in D_1 \oplus \langle \xi \rangle$, by equation (2.18)

$$(\nabla_X N)X = -h(X, TX) + nh(X, X)$$

or

$$N\nabla_X X = nh(X, X). \tag{4.2}$$

Now, making use of proposition (3.2) and the assumption that $h(X, TX) = 0$, we get $h(X, \phi Y) = 0$ i.e., $h(X, Y) = 0$ for each $X, Y \in D_1 \oplus \langle \xi \rangle$. This proves that leaves of $D_1 \oplus \langle \xi \rangle$ are totally geodesic in $\bar{M}$. In particular $h(X, X) = 0$. Thus by (4.2) we obtain that $\nabla_X X \in D_1 \oplus \langle \xi \rangle$. This fact together with integrability of $D_1 \oplus \langle \xi \rangle$ implies that $\nabla_X Y \in D_1 \oplus \langle \xi \rangle$ i.e., leaves of $D_1 \oplus \langle \xi \rangle$ are totally geodesic in M. □

As an immediate consequence of the above, we have.

Corollary 4.1. *Let M be a totally umbilical semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$. If $D_1 \oplus \langle \xi \rangle$ is integrable then leaves of $D_1 \oplus \langle \xi \rangle$ are totally geodesic in M as well as in $\bar{M}$.*

Theorem 4.1. *Let M be a totally umbilical semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$, with dim $D_1 \neq 0$, then the mean curvature vector field H of M is a global section of ND_2.*

Proof. Let $X \in D_1$ be a unit vector field and $V \in \mu$, then

$$g(H,V) = g(h(X,X),V) = g(\bar{\nabla}_X \phi X, \phi V) = g(h(X,\phi X),\phi V) = 0.$$

which proves the assertion. □

Theorem 4.2. *Let M be a totally umbilical semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$ with $\alpha \neq 0$ on M. Then M is an invariant submanifod.*

Proof. By (4.1) $h(Z,\xi) = 0$, for any $Z \in D_2$ and therefore by (2.16)(b) $NZ = 0$. Hence, M is an invariant submanifold.

Consequently, we may state: □

Corollary 4.2. *There does not exist totally umbilical proper semi-slant submanifold of a trans-Sasakian manifold provided $\alpha \neq 0$ on M.*

From theorems (4.1) and (4.2), it follows that

Corollary 4.3. *A totally umbilical semi-slant submanifold of a trans-Sasakian manifold $\bar{M}$, is in fact totally geodesic in $\bar{M}$.*

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Department of Mathematics
Aligarh Muslim University
Aligarh, India 20 2002
E-mail: viqar@math.com
E-mail: ali_mrj@cyberamu.com