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ON WEAKLY SEMI-I-OPEN SETS AND ANOTHER DECOMPOSITION OF CONTINUITY VIA IDEALS

ESREF HATIR AND SAEID JAFARI

Abstract. In this paper, we introduce the notions of weakly semi-I-open set and weakly semi-I-continuous functions to obtain a decomposition of continuity. We also investigated the fundamental properties of such functions.

1. Introduction

Topological ideals have played an important role in topology for several years. It was the works of Newcomb [13], Rancin [14], Samuels [15] and Hamlet and Jankovic [4, 5, 6, 7, 10] which motivated the research in applying topological ideals to generalize the most basic properties in general topology. In 1992, Jankovic and Hamlett [7] introduced the notion of I-open sets in topological spaces. El-monsef et al. [2] investigated I-open sets and I-continuous functions. In 1996, Dontchev [3] introduced the notion of pre-I-open sets and obtained a decomposition of I-continuity. Quite recently, Hatir and Noiri [8] have introduced the notion of semi-I-open sets to obtain another new decomposition of continuity.

In this paper, we introduce the notions of weakly semi-I-open set and weakly semi-I-continuous function to obtain a decomposition of continuity via ideals. We also investigate the fundamental properties of such functions.

Throughout this paper, $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ denote the closure of A and the interior of A, respectively. Let (X, τ) be a topological space and I an ideal of subsets of X. An *ideal topological space*, denoted by (X, τ, I), is a topological space (X, τ) with an ideal I on X. For a subset $A \subset X$, $A^*(I) = \{x \in X : U \cap A \notin I$ for each neighborhood U of $x\}$ is called the *local function* [3] of A with respect to I and τ. We simply write A^* instead of $A^*(I)$ in case there is no chance for confusion. X^* is often a proper subset of X. The hypothesis $X = X^*$ [9] is equivalent to the hypothesis $\tau \cap I = \{\emptyset\}$ [15]. For every

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ideal topological space (X, τ, I), there exists a topology $\tau^*(I)$, finer than τ, generated by the base $\beta(I, \tau) = \{U \backslash G : U \in \tau \text{ and } G \in I\}$. However, $\beta(I, \tau)$ is not always a topology [10]. Recall that $\mathrm{Cl}^*(A) = A \cup A^*$ defines a Kuratowski closure operator. In what follows the space (X, τ, I) is always taken to be an ideal topological space.

Definition 1.1. *A subset S of a space (X, τ, I) is said to be*

a) *I-open* [2] *if* $S \subset \mathrm{Int}(S^*)$,
b) *pre-I-open* [3] *if* $S \subset \mathrm{Int}(\mathrm{Cl}^*(S))$,
c) *semi-I-open* [8] *if* $S \subset \mathrm{Cl}^*(\mathrm{Int}(S))$.
d) *α-I-open* [8] *if* $S \subset \mathrm{Int}(\mathrm{Cl}^*(\mathrm{Int}(S)))$.

Definition 1.2. *A subset S of a space (X, τ) is said to be*

a) *pre-open* [12] *if* $S \subset \mathrm{Int}(\mathrm{Cl}(S))$,
b) *semi-open* [11] *if* $S \subset \mathrm{Cl}(Int(S))$,
c) *β-open* [1] *if* $S \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(S)))$.

Hatir and Noiri [8] obtained the following diagram:

$$\begin{array}{ccccccc} \text{Open} & \rightarrow & \alpha\text{-I-open} & \rightarrow & \text{semi-I-open} & \rightarrow & \text{semi-open} \\ & & \downarrow & & & & \downarrow \\ \text{I-open} & \rightarrow & \text{pre-I-open} & \rightarrow & \text{pre-open} & \rightarrow & \beta\text{-open.} \end{array}$$

2. Weakly semi-I-open sets and another decomposition of continuity

Definition 2.1. *A subset S of a space (X, τ, I) is said to be weakly semi-I-open if* $S \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(S)))$.

Remark 2.1. Every semi-I-open set is weakly semi-I-open set, but not conversely.

Example 2.1. Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a, b\}\}$ and $I = \{\emptyset, \{c\}\}$. Then $A = \{a\}$ is weakly semi-I-open, but not semi-I-open.

Remark 2.2. Every weakly semi-I-open set is β-open set, but not conversely.

Let S be a weakly semi-I-open set i.e. $S \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(S)))$. Then $S \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(S))) = (\mathrm{Int}(\mathrm{Cl}(S)))^* \cup (\mathrm{Int}(\mathrm{Cl}(S))) \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(S))) \cup \mathrm{Int}(\mathrm{Cl}(S)) = \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(S)))$. However:

Example 2.2. Let X={a,b,c}, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{a, c\}$ is β-open, but not weakly semi-I-open.

By WSIO(X, τ), we denote the family of all weakly semi-I-open sets of space (X, τ, I).

Lemma 2.1. ([10].) *Let A and B be subsets of a space (X,τ,I). Then*

1) *If $A \subset B$, then $A^* \subset B^*$,*
2) *If $U \in \tau$, then $U \cap A^* \subset (U \cap A)^*$.*
3) *A^* is closed in (X,τ).*

Theorem 2.1. *Let (X,τ,I) be an ideal topological space and A, B subsets of X.*

1) *If $U_\alpha \in \mathrm{WSIO}\,(X,\tau)$ for each $\alpha \in \Delta$, then $\cup\{U_\alpha : \alpha \in \Delta\} \in \mathrm{WSIO}\,(X,\tau)$,*
2) *If $A \in \mathrm{WSIO}(X,\tau)$ and $B \in \tau$, then $A \cap B \in \mathrm{WSIO}(X,\tau)$.*

Proof. (1). Since $U_\alpha \in \mathrm{WSIO}\,(X,\tau)$, we have $U_\alpha \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(U_{\alpha)}))$ for every $\alpha \in \Delta$. Thus by using Lemma 2.1, we obtain
$\cup_{\alpha\in\Delta}U_\alpha \subset \cup_{\alpha\in\Delta}\mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(U_\alpha))) \subset \cup_{\alpha\in\Delta}\{(\mathrm{Int}(\mathrm{Cl}(U_{\alpha)}))^* \cup (\mathrm{Int}(\mathrm{Cl}(U_\alpha))\} \subset (\cup_{\alpha\in\Delta}(\mathrm{Int}\,(\mathrm{Cl}\,(U_\alpha))))^* \cup \mathrm{Int}\,(\mathrm{Cl}\,(\cup_{\alpha\in\Delta}U_\alpha)) \subset (\mathrm{Int}\,(\mathrm{Cl}\,(\cup_{\alpha\in\Delta}U_\alpha)))^* \cup \mathrm{Int}\,(\mathrm{Cl}(\cup_{\alpha\in\Delta}U_\alpha)) = \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(\cup_{\alpha\in\Delta}U_\alpha)))$.

This shows that $\cup_{\alpha\in\Delta}U_\alpha \in \mathrm{WSIO}\,(X,\tau)$.

(2). Let $A \in \mathrm{WSIO}\,(X,\tau)$ and $B \in \tau$, then $A \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(A)))$ and by Lemma 2.1, we obtain
$A\cap B \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(A)))\cap B = ((\mathrm{Int}(\mathrm{Cl}(A)))^* \cup \mathrm{Int}(\mathrm{Cl}(A)))\cap B = (\mathrm{Int}(\mathrm{Cl}(A)))^* \cap B \cup \mathrm{Int}(\mathrm{Cl}(A)) \cap B \subset (\mathrm{Int}(\mathrm{Cl}(A)) \cap B^*) \cup \mathrm{Int}(\mathrm{Cl}(A \cap B)) = (\mathrm{Int}(\mathrm{Cl}(A \cap B)))^* \cup \mathrm{Int}(\mathrm{Cl}(A \cap B)) = \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(A \cap B)))$.

This shows that $A \cap B \in \mathrm{WSIO}\,(X,\tau)$. □

Remark 2.3. A finite intersection of weakly semi-I-open sets need not be weakly semi-I-open in general as the following example shows.

Example 2.3. Let $X = \{a,b,c,d\}, \tau = \{\emptyset, X, \{a,b\}, \{a,b,c\}\}$ and $I = \{\emptyset, \{c\}, \{d\}, \{c,d\}\}$. Then $A = \{b,d\}$ and $B = \{a,d\}$ is weakly semi-I-open, but $A \cap B$ is not weakly semi-I-open.

Definition 2.2. *A subset F of a space (X,τ,I) is said to be weakly semi-I-closed if its complement is weakly semi-I-open.*

Theorem 2.2. *A subset A of a space (X,τ,I) is weakly semi-I-closed if and only if* $\mathrm{Int}^*(\mathrm{Cl}(\mathrm{Int}(A))) \subset A$.

Proof. Let A be a weakly semi-I-closed set of (X,τ,I). Then $X - A$ is weakly semi-I-open and hence $X - A \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(X-A))) = X - Int^*(\mathrm{Cl}(\mathrm{Int}(A)))$. Therefore, we have $\mathrm{Int}^*(\mathrm{Cl}(\mathrm{Int}(A))) \subset A$.

Conversely, let $\mathrm{Int}^*(\mathrm{Cl}(\mathrm{Int}(A))) \subset A$. Then $X - A \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(X - A)))$ and hence $X - A$ is weakly semi-I-open. Therefore, A is weakly semi-I-closed. □

Remark 2.4. For a subset A of a space (X,τ,I), we have $X - \mathrm{Int}(\mathrm{Cl}^*(\mathrm{Int}(A))) \neq \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(X - A)))$ as shown by the following example.

Example 2.4. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{b\}, \{a, b\}\}$. If we put $A = \{a, c\}$, then $X - \mathrm{Int}(\mathrm{Cl}^*(\mathrm{Int}(A))) = \{b, c\}$ and $\mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(X - A))) = \{b\}$.

Theorem 2.3. *If a subset A of a space (X, τ, I) is weakly semi-I-closed, then* $\mathrm{Int}(\mathrm{Cl}^*(\mathrm{Int}(A))) \subset A$.

Proof. Let A be any weakly semi-I-closed set of (X, τ, I). Since $\tau^*(I)$ is finer than τ, we have $\mathrm{Int}(\mathrm{Cl}^*(\mathrm{Int}(A))) \subset \mathrm{Int}^*(\mathrm{Cl}^*(\mathrm{Int}(A))) \subset \mathrm{Int}^*(\mathrm{Cl}(\mathrm{Int}(A)))$. Therefore, by Theorem 2.2, we obtain $\mathrm{Int}(\mathrm{Cl}^*(\mathrm{Int}(A))) \subset A$. □

Corollary 2.1. *Let A be a subset of a space (X, τ, I) such that* $X - \mathrm{Int}(\mathrm{Cl}^*(\mathrm{Int}(A))) = \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(X - A)))$. *Then A is weakly semi-I-closed if and only if* $\mathrm{Int}(\mathrm{Cl}^*(\mathrm{Int}(A))) \subset A$.

Proof. This is an immediate consequence of Theorem 2.2. □

Definition 2.3. *A subset A of a space (X, τ, I) is called*

a) *Strong S-I-set if* $\mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(A))) = \mathrm{Int}(A)$,
b) *S-I-set* [8] *if* $\mathrm{Cl}^*(\mathrm{Int}(A)) = \mathrm{Int}(A)$.

Remark 2.5. Observe that in Example 2.1, the set $A = \{a\}$ is not a strong S-I-set. If we set $X = \{a, b, c\}, \tau = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $I = \{\emptyset, \{c\}\}$, then $A = \{a\}$ is a strong S-I-set. In Example 2.2 [8], the set of rationals Q is S-I-set but not a strong S-I-set.

Definition 2.4. *A subset of an ideal topological space (X, τ, I) is called*

a) *Strong S_I-set if $A = U \cap V$, where $U \in \tau$ and V is strong S-I-set,*
b) *S_I-set* [8] *if $A = U \cap V$, where $U \in \tau$ and V is S-I-set.*

Remark 2.6. a) Every strong S-I-set is S-I-set,

b) Every strong S_I-set is S_I-set,
c) Every open set is strong S_I-set.

Proposition 2.1. *For a subset A of (X, τ, I), the following conditions are equivalent:*

a) *A is open,*
b) *A is weakly semi-I-open and strong S_I-set,*
c) *A is semi-I-open and S_I-set* [8].

Proof. By the above remarks is only necessary to show the following:

If A is a weakly semi-I-open set and also a strong S_I-set, then $A \subseteq \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(A))) = \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(U \cap V)))$, where $U \in \tau$ and V is strong S-I-set. Hence $A \subset U \cap A \subset U \cap (\mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(U))) \cap \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(V)))) = U \cap \mathrm{Int}(V) = Int(A)$.

This shows that A is open. □

Definition 2.5. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be weakly semi-I-continuous (resp. semi-I-continuous* [8]*, strong S_I-continuous) if $f^{-1}(V)$ is weakly semi-I-open (resp. semi-I-open, strong S_I-open) in (X, τ, I) for every open set V of (Y, σ).*

Remark 2.7. It is obvious that continuity implies semi-I-continuity and semi-I-continuity implies weakly semi-I-continuity and weakly semi-I-continuity implies β-continuity.

Theorem 2.4. *For a function $f : (X, \tau, I) \to (Y, \sigma)$ the following conditions are equivalent:*

1) *f is continuous,*
2) *f is weakly semi-I-continuous and strong S_I-continuous ,*
3) *f is semi-I-continuous and S_I-continuous* [8].

Proof. It is obvious from Proposition 2.1. □

Theorem 2.5. *For a function $f : (X, \tau, I) \to (Y, \sigma)$, the following are equivalent:*

1) *f is weakly semi-I-continuous,*
2) *For each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists $U \in \mathrm{WSIO}\,(X, \tau)$ containing x such that $f(U) \subset V$,*
3) *The inverse image of each closed set in Y is weakly semi-I-closed.*

Proof. Straightforward. □

Definition 2.6. *f is weakly I-irresolute, if $f^{-1}(V)$ is weakly semi-I-open in (X, τ, I) for every weakly semi-I-open set V of (Y, σ, J).*

Theorem 2.6. *The following hold for a functions $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \eta)$,*

1) *$g \circ f$ is weakly semi-I-continuous if f is weakly semi-I-continuous and g is continuous,*
2) *$g \circ f$ is weakly semi-I-continuous if f is weakly I-irresolute and g is weakly semi-I-continuous.*

If (X, τ, I) is an ideal topological space and A is subset of X, we denote by $\tau_{|A}$ the relative topology on A and $I_{|A} = \{A \cap I \mid I \in I\}$ is obviously an ideal on A.

Lemma 2.2. ([10]). *Let (X, τ, I) be an ideal topological space and B, a subsets of X such that $B \subset A$. Then $B^*(\tau_{|A}, I_{|A}) = B^*(\tau, I) \cap A$.*

Lemma 2.3. *Let (X, τ, I) be an ideal topological space, $A \subset X$ and $U \in \tau$. Then $\mathrm{Cl}^*(A) \cap U = \mathrm{Cl}^*(A \cap U)$.*

Proof. $\mathrm{Cl}^*(A) \cap U = (A^* \cup A) \cap U = (A^* \cap U) \cup (A \cap U) \subset (A \cap U)^* \cup (A \cap U) = \mathrm{Cl}^*(A \cap U)$. □

Theorem 2.7. *Let (X, τ, I) be an ideal topological space. If $U \in \tau$ and $A \in \mathrm{WSIO}\,(X, \tau)$, then $U \cap A \in \mathrm{WSIO}\,(U, \tau_{|U}, I_{|U})$.*

Proof. Since $U \in \tau$ and $A \in \mathrm{WSIO}\,(X, \tau)$, by Theorem 2.1, we have $A \cap U \subset \mathrm{Cl}^*(\mathrm{Int}(\mathrm{Cl}(A \cap U))$ and hence by Lemma 2.2, $A \cap U \subset U \cap \mathrm{Cl}^*(\mathrm{Int}(Cl(A \cap U))) \subset \mathrm{Cl}^*(U \cap \mathrm{Int}(\mathrm{Cl}(A \cap U))) \subset \mathrm{Cl}^*(\mathrm{Int}[U \cap \mathrm{Cl}(A \cap U)]) = \mathrm{Cl}^*(\mathrm{Int}_U(U \cap \mathrm{Cl}(A \cap U)))$. Since $U \in \tau \subset \tau^*$, we obtain

$$A \cap U \subset U \cap \mathrm{Cl}^*(\mathrm{Int}_U(\mathrm{Cl}_U(A \cap U))) = \mathrm{Cl}^*_U(\mathrm{Int}_U(\mathrm{Cl}_U(A \cap U))).$$

This shows that $A \cap U \in \mathrm{WSIO}\,(U, \tau_{|U}, I_{|U})$. □

Theorem 2.8. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be weakly semi-I-continuous function and $U \in \tau$. Then the restriction $f_{|U} : (U, \tau_{|U}, I_{|U}) \to (Y, \sigma)$ is weakly semi-I-continuous.*

Proof. Let V be any open set of (Y, σ). Since f is weakly semi-I-continuous, we have $f^{-1}(V) \in \mathrm{WSIO}\,(X, \tau)$. Since $U \in \tau$, by Theorem 2.7, we have $U \cap f^{-1}(V) \in \mathrm{WSIO}\,(U, \tau_{|U}, I_{|U})$. On the other hand, $(f_{|U})^{-1}(V) = U \cap f^{-1}(V)$ and $(f_{|U})^{-1}(V) \in \mathrm{WSIO}\,(U, \tau_{|U}, I_{|U})$. This shows that $f_{|U} : (U, \tau_{|U}, I_{|U}) \to (Y, \sigma)$ is weakly semi-I-continuous. □

Theorem 2.9. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is weakly semi-I-continuous if and only if the graph function $g : X \to XxY$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is weakly semi-I-continuous.*

Proof. Necessity. Suppose that f is weakly semi-I-continuous. Let $x \in X$ and W be any open set of XxY containing $g(x)$. Then there exists a basic open set UxV such that $g(x) = (x, f(x)) \in UxV \subset W$. Since f is weakly semi-I-continuous, there exists a weakly semi-I-open set U_o of X containing x such that $f(U_o) \subset V$. By Theorem 2.7, $U_o \cap U \in \mathrm{WSIO}\,(X, \tau)$ and $g(U_o \cap U) \subset UxV \subset W$. This shows that g is weakly semi-I-continuous.

Sufficiency. Suppose that g is weakly semi-I-continuous. Let $x \in X$ and V be any open set of Y containing $f(x)$. Then XxV is open in XxY and by weakly semi-I-continuity of g, there exists $U \in \mathrm{WSIO}\,(X, \tau)$ containing x such that $g(U) \subset XxV$. Therefore, we obtain $f(U) \subset V$. This shows that f is weakly semi-I-continuous. □

3. Weakly semi-I-open and weakly semi-I-closed functions

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma, J)$ is called weakly semi-I-open (resp. weakly semi-I-closed) if the image of every open set (resp. closed) in (X, τ) is weakly semi-I-open (resp. weakly semi-I-closed) in (Y, σ, J).*

Recall that a function $f : (X, \tau) \to (Y, \sigma)$ is called *β-open* (resp. *β-closed*) if the image of each open (resp. closed) set in X is β-open (resp. β-closed) in Y. Also a function $f : (X, \tau) \to (Y, \sigma)$ is called *semi-I-open* (resp. *semi-I-closed*) if the image of each open (resp. closed) set in X is semi-I-open (resp. semi-I-closed) in Y.

Clearly every weakly semi-I-open (resp. weakly semi-I-closed) function is β-open (resp. β-closed) but the converse is not true in general.

Observe that every semi-I-open (resp. semi-I-closed) function is weakly semi-I-open (resp. weakly semi-I-closed) but the converse is not true in general.

Now we have the following results. The proofs of them are omitted since they are straightforward.

Theorem 3.1. *A function $f : (X, \tau) \to (Y, \sigma, J)$ is weakly semi-I-open if and only if for each point x of X and each neighborhood U of x, there exists a weakly semi-I-open set V in Y containing $f(x)$ such that $V \subset f(U)$.*

Theorem 3.2. *If $f : (X, \tau) \to (Y, \sigma, J)$ is weakly semi-I-open (resp. weakly semi-I-closed) function such that $D \subset Y$ and $E \subset X$ is a closed (resp. open) set containing $f^{-1}(D)$, then there exists a weakly semi-I-open (resp. weakly semi-I-closed) set $M \subset Y$ containing D such that $f^{-1}(M) \subset E$.*

Theorem 3.3. *For any bijective function $f : (X, \tau) \to (Y, \sigma, J)$ the following conditions hold:*

(1) *The inverse function is weakly semi-I-continuous;*
(2) *f is weakly semi-I-open;*
(3) *f is weakly semi-I-closed.*

Theorem 3.4. *Let $f : (X, \tau) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \nu, K)$ be two functions, where I, J and K are ideals on X, Y and Z, respectively. The following statements hold:*

(1) *$g \circ f$ is weakly semi-I-open, if f is open and g is weakly semi-I-open.*
(2) *f is weakly semi-I-open if $g \circ f$ is open and g is weakly semi-I-continuous injection.*

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E. Hatir
Selçuk Üniversitesi
Eğitim Fakültesi
Matematik Bölümü
42090 Konya, Turkey
E–mail: hatir10@yahoo.com

S. Jafari
College of Vestsjaelland South
Herrestraede 11
4200 Slagelse, Denmark
E–mail: jafari@stofanet.dk