SARAJEVO JOURNAL OF MATHEMATICS
Vol.1 (14) (2005), 171–174

ON $(\mathbf{n},\mathbf{m})-$GROUPS FOR $\mathbf{n}>\mathbf{2m}$

RADOSLAV GALIĆ

Abstract. In this article a theorem about a $(2m,m)-$group of Čupona-Dimovski is generalized.

1. Preliminaries

Definition 1.1. [1] *Let $n\geq m+1$ and let $(Q;A)$ be an $(n,m)-$groupoid $(A:Q^n\to Q^m)$. We say that $(Q;A)$ is an $(n,m)-$group iff the following statements hold:*

(i) *For every $i,j\in\{1,\dots,n-m+1\}$, $i<j$, the following law holds*

$$A(x_1^{i-1},A(x_i^{i+n-1}),x_{i+n}^{2n-m})=A(x_1^{j-1},A(x_j^{j+n-1}),x_{j+n}^{2n-m})$$

$[:<i,j>$ *−associative law*$]$; *and*

(ii) *For every $i\in\{1,\dots,n-m+1\}$ and for every $a_1^n\in Q$ there is exactly one $x_1^m\in Q^m$ such that the following equality holds*

$$A(a_1^{i-1},x_1^m,a_i^{n-m})=a_{n-m+1}^n.$$

Also see [3].

Definition 1.2. [6] *Let $n\geq 2m$ and let $(Q;A)$ be an $(n,m)-$groupoid. Let also $\mathbf{e}$ be a mapping of the set Q^{n-2m} into the set Q^m. Then, we say that $\mathbf{e}$ is a $\{1,n-m+1\}-$**neutral operation of the** $(n,m)-$**groupoid** $(Q;A)$ iff for every sequence a_1^{n-2m} over Q and for every $x_1^m\in Q^m$ the following equalities hold*

$$A(x_1^m,a_1^{n-2m},\mathbf{e}(a_1^{n-2m}))=x_1^m \text{ and } A(\mathbf{e}(a_1^{n-2m}),a_1^{n-2m},x_1^m)=x_1^m.$$

Remark. For $m=1$ $\mathbf{e}$ is a $\{1,n\}-$neutral operation of the $n-$groupoid $(Q;A)$ [5]. Cf. Chapter II in [8].

2000 *Mathematics Subject Classification.* Primary: 20N15.
Key words and phrases. $(n,m)-$group, $\{1,n-m+1\}-$neutral operation of the $(n,m)-$groupoid.

Proposition 1.3. [6] *Let* $(Q; A)$ *be an* $(n, m)-$*groupoid and let* $n \geq 2m$. *Then there is at most one* $\{1, n-m+1\}-$*neutral operation of* $(Q; A)$.

Proposition 1.4. [6] *Every* $(n, m)-$*group* $(n \geq 2m)$ *has a* $\{1, n-m+1\}-$*neutral operation.*

Also see [7].

2. Auxiliary proposition

Proposition 2.1. [4] *Let* $(Q; A)$ *be an* $(n, m)-$*group,* $\mathbf{e}$ *its* $\{1, n-m+1\}-$*neutral operation* (1.2–1.4) *and* $n > 2m$. *Then for every* $a_1^{n-2m}, x_1^m \in Q$ *and for all* $j \in \{1, \ldots, n-2m+1\}$ *the following equalities hold*

$$A(x_1^m, a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}) = x_1^m \quad \textit{and} \tag{1}$$

$$A(a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}, x_1^m) = x_1^m. \tag{2}$$

Remark. For $m = 1$ see Proposition 1.1–IV in [8].

Main part of the proof.

$F(x_1^m, a_1^{n-2m}) \overset{def}{=} A(a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}, x_1^m) \Rightarrow$
$A(a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}, F(x_1^m, a_1^{n-2m})) =$
$A(a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}, A(a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}, x_1^m)) \overset{(i)}{\Longrightarrow}$
$A(a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}, F(x_1^m, a_1^{n-2m})) =$
$A(a_j^{n-2m}, A(\mathbf{e}(a_1^{n-2m}), a_1^{n-2m}, \mathbf{e}(a_1^{n-2m})), a_1^{j-1}, x_1^m) \overset{1.2,1.4}{\Longrightarrow}$
$A(a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}, F(x_1^m, a_1^{n-2m})) =$
$A(a_j^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{j-1}, x_1^m) \overset{(ii)}{\Longrightarrow} F(x_1^m, a_1^{n-2m}) = x_1^m.$

Whence, we obtain (2). □

3. Result

Theorem 3.1. *Let* $n > 2m, m > 1$, $(Q; A)$ *be an* $(n, m)-$*group and* $\mathbf{e}$ *its* $\{1, n-m+1\}-$*neutral operation* (1.2-1.4). *Then for all* $i \in \{0, 1, \ldots, m\}$ *for all* $t \in \{1, \ldots, n-2m+1\}$, *for every* $x_1^m \in Q^m$ *and for every sequence* a_1^{n-2m} *over* Q *the following equality holds*

$$A(x_1^i, a_t^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{t-1}, x_{i+1}^m) = x_1^m.$$

Remark. Theorem 3.1 for $n = 2m$ $(m > 1)$ is proved in [2]. Also see [3].

Main part of the proof.

1) Instead of $\mathbf{e}(a_1^{n-2m})$ we write

$$\overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=1}^{m}$$

where $\mathbf{e}_j : Q^{n-2m} \to Q$.

2)

$$A(x_1^i, a_t^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{t-1}, x_{i+1}^m) \overset{(2)j=1}{=\!=}$$

$$A(a_1^{n-2m}, \mathbf{e}(a_1^{n-2m}), A(x_1^i, a_t^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{t-1}, x_{i+1}^m)) \overset{1)}{=}$$

$$A(a_1^{n-2m}, \overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=1}^{m}, A(x_1^i, a_t^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{t-1}, x_{i+1}^m)) =$$

$$A(a_1^{n-2m}, \overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=1}^{i}, \overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=i+1}^{m},$$

$$A(x_1^i, a_t^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{t-1}, x_{i+1}^m)) \overset{(i)}{=}$$

$$A(a_1^{n-2m}, \overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=1}^{i}, A(\overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=i+1}^{m},$$

$$x_1^i, a_t^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{t-1}), x_{i+1}^m) \overset{(1)}{=}$$

$$A(a_1^{n-2m}, \overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=1}^{i}, \overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=i+1}^{m}, x_1^i, x_{i+1}^m) =$$

$$A(a_1^{n-2m}, \overline{\mathbf{e}_j(a_1^{n-2m})}\,\Big|_{j=1}^{m}, x_1^m) \overset{1)}{=}$$

$$A(a_1^{n-2m}, \mathbf{e}(a_1^{n-2m}), x_1^m) \overset{(2)j=1}{=\!=} x_1^m,\ 0 < i < m.$$

□

References

[1] Ǵ. Čupona, *Vector valued semigroups*, Semigroup Forum 26 (1983), 65–74.

[2] Ǵ. Čupona and D. Dimovski, *On a class of vector valued groups*, Proceedings of the Conf. "Algebra and Logic", Zagreb 1984, 29–37.

[3] Ǵ. Čupona, N. Celakoski, S. Markovski and D. Dimovski, *Vector valued groupoids, semigroups and groups*, in: Vector valued semigroups and groups, (B. Popov, Ǵ. Čupona and N. Celakoski, eds.), Skopje 1988, 1–78.

[4] R. Galić and A. Katić, *On neutral operations of* $(n,m)-$*groups*, 2004, notes.

[5] J. Ušan, *Neutral operations of* $n-$*groupoids*, (Russian), Rev. of Research, Fac. of Sci. Univ. of Novi Sad, Math. Ser., 18 (2) (1988), 117–126.

[6] J. Ušan, *Neutral operations of* $(n,m)-$*groupoids*, (Russian), Rev. of Research, Fac. of Sci. Univ. of Novi Sad, Math. Ser., 19 (2) (1989), 125–137.

[7] J. Ušan, *Note on* $(n,m)-$*groups*, Math. Moravica, 3 (1999), 127–139.

[8] J. Ušan, *n-groups in the light of the neutral operations*, Math. Moravica, Special Vol., (2003), monograph.

(Received: March 30, 2005)

Faculty of Electrical Engineering
University of Osijek
Kneza Trpimira 2B, HR - 31000 Osijek
Croatia