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# ON SOME DOUBLE WEIGHTED FRACTIONAL INTEGRAL INEQUALITIES

MOHAMED BEZZIOU, ZOUBIR DAHMANI AND AMINA KHAMELI

Abstract. In this paper, we establish new double weighted results of Chebyshev inequalities involving the Riemann-Liouville fractional integral operator with respect to another measurable, increasing, positive and monotone function. In particular, some recent results are deduced as some special cases.

## 1. Introduction

We consider the double weighted Chebyshev functionals [3, 4]:

$$T(f,g,p) := \int_a^b p(x)dx \int_a^b p(x)f(x)g(x)dx - \int_a^b p(x)f(x)dx \int_a^b p(x)g(x)dx,$$

and

$$\begin{aligned} T(f,g,p,q) &:= \int_a^b p(x)dx \int_a^b q(x)f(x)g(x)dx \\ &+ \int_a^b q(x)dx \int_a^b p(x)f(x)g(x)dx - \int_a^b p(x)f(x)dx \int_a^b q(x)g(x)dx \\ &- \int_a^b q(x)f(x)dx \int_a^b p(x)g(x)dx, \end{aligned} \tag{1.1}$$

where $f$ and $g$ are two integrable functions on $[a,b]$ and $p$ and $q$ are positive integrable functions on $[a,b]$. If $p=q$ then $T(f,g,p)=\frac{1}{2}T(f,g,p,q)$, and if $f$ and $g$ are synchronous on $[a,b]$, then we have $T(f,g,p)\geq 0, T(f,g,p,q)\geq 0$. Recently, K.M Awan et al. proved the following very intersting result [1]: If $\varphi$ is an absolutely continuous function on $[a,b]$ and $p$ is a positive and integrable function on $[a,b]$, with $(\varphi')^2 \in L^1[a,b]$, then it yields that:

$$T(\varphi,\varphi,p) \leq \frac{1}{P^2(b)} \int_a^b \left[\int_a^x p(t)dt \int_a^b tp(t)dt - \int_a^b p(t)dt \int_a^x tp(t)dt\right] \left[\varphi'(x)\right]^2 dx,$$

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$$P(b)=\int_a^b p(t)dt.$$

Very recently, M. Bezziou et al. established the following result [2]:

**Theorem 1.1.** *Suppose that $\phi:[a,b]\to\mathbb{R}$ is an absolutely continuous function and $p:[a,b]\to\mathbb{R}^+$ is an integrable function. If $(\phi')^2\in L^1[a,b]$, then for all $\alpha>0$, we have*

$$I_a^\alpha p(b) I_a^\alpha p\phi^2(b)-\left(I_a^\alpha p\phi(b)\right)^2\leq\int_a^b H(x)\left[\phi'(x)\right]^2 dx,$$

*with*

$$H(x):=\frac{1}{2}\left[I_a^\alpha bp(b)\int_a^x (b-t)^{\alpha-1}p(t)dt\ -I_a^\alpha p(b)\int_a^x t(b-t)^{\alpha-1}p(t)dt\right],$$

*where $I_a^\alpha$ is the Riemann-Liouville fractional integral.*

Then, using another method, Z. Dahmani and M. Doubbi Bounoua proved the result [7]:

**Theorem 1.2.** *Let $\phi:[a,b]\to\mathbb{R}$ be an absolutely continuous function, $p:[a,b]\to\mathbb{R}$ an integrable function and $(\phi')^2\in L^1[a,b]$. Then, for all $\alpha>0$ and $x\in]a,b]$, we have:*

$$\frac{1}{I_a^\alpha p(x)}I_a^\alpha(p\phi^2)(x)-\Big[\frac{1}{I_a^\alpha p(x)}I_a^\alpha(p\phi)(x)\Big]^2\leq\frac{1}{[I_a^\alpha p(x)]^2}\int_a^x\widetilde{P}_x(t)[\phi'(t)]^2dt,$$

*with*

$$\widetilde{P}_x(t)=\frac{1}{\Gamma(\alpha)}\left[I_a^\alpha(xp(x))\int_a^t p(y)(x-y)^{\alpha-1}dy-I_a^\alpha p(x)\int_a^t yp(y)(x-y)^{\alpha-1}dy\right],$$

*where $I_a^\alpha$ is the Riemann-Liouville fractional integral.*

Many researchers have been concerned with the functionals (1.1) and (1.1). For more details, we refer the reader to [8, 9, 10, 11, 13, 14] and the references therein.
The main purpose of this paper is to establish some new inequalities for (1.1) by using the Riemann-Liouville fractional integrals with respect to another measurable, increasing, positive and monotone function $h$. Our results allow us to generalize some very recent results of the paper [2].

## 2. Preliminaries

We recall two definitions and some properties that will be used in this paper. We have:

**Definition 2.1.** ([6]) The Riemann-Liouville fractional integral operator of order $\alpha\geq 0$, for an integrable function $f$ defined on $[a,b]$ is given by

$$\begin{aligned} I_a^\alpha f(t) &= \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau,\ \alpha > 0,\ a < t \le b, \\ I_a^0 f(t) &= f(t). \end{aligned}$$

We have, for $\alpha > 0$, $\beta > 0$, the following property

$$I_a^\alpha I_a^\beta f(t) = I_a^{\alpha+\beta} f(t),$$

which implies

$$I_a^\alpha I_a^\beta f(t) = I_a^\beta I_a^\alpha f(t).$$

**Definition 2.2.** ([12]) The Riemann-Liouville fractional integral operator of order $\alpha \ge 0$, for an integrable function $f$ defined on $[a,b]$, with respect to another measurable, increasing, positive and monotone function $h$ on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ is defined as

$$I_{a,h}^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (h(t) - h(\tau))^{\alpha-1} h'(\tau) f(\tau)\, d\tau,\ \alpha > 0,\ a < t \le b.$$

*Remark* 2.3. If we take $h(x) = x$ in the Definition 2.2, we obtain Definition 2.1.

## 3. Main Results

We begin by proving the following result:

**Lemma 3.1.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$, $g : [a,b] \to \mathbb{R}$ is a continuous function on $[a,b]$ and $p, q : [a,b] \to \mathbb{R}^+$ are positive integrable functions. Then, for all $\alpha > 0$, we have*

$$\begin{aligned} & \left[I_{a,h}^\alpha p(b)\right] \left[I_{a,h}^\alpha b\,(qg)(b)\right] + \left[I_{a,h}^\alpha q(b)\right] \left[I_{a,h}^\alpha b\,(pg)(b)\right] \\ & - \left[I_{a,h}^\alpha bp(b)\right] \left[I_{a,h}^\alpha (qg)(b)\right] - \left[I_{a,h}^\alpha bq(b)\right] \left[I_{a,h}^\alpha (pg)(b)\right] \qquad (3.1) \\ = & \frac{1}{\Gamma(\alpha)} \int_a^b \left[ I_{a,h}^\alpha bp(b) \int_a^x (h(b) - h(t))^{\alpha-1} h'(t)\, q(t) dt \right. \\ & \left. - I_{a,h}^\alpha p(b) \int_a^x t (h(b) - h(t))^{\alpha-1} h'(t)\, q(t) dt \right] \left(g'(x)\right) dx. \end{aligned}$$

*Proof.* Suppose that $f : [a,b] \to \mathbb{R}$ is a continuous function on $[a,b]$. We can write

$$\begin{aligned} & \left[I_{a,h}^\alpha p(b)\right] \left[I_{a,h}^\alpha (qfg)(b)\right] + \left[I_{a,h}^\alpha q(b)\right] \left[I_{a,h}^\alpha (pfg)(b)\right] \\ & - \left[I_{a,h}^\alpha (pf)(b)\right] \left[I_{a,h}^\alpha (qg)(b)\right] - \left[I_{a,h}^\alpha (qf)(b)\right] \left[I_{a,h}^\alpha (pg)(b)\right] \\ = & \frac{1}{\Gamma^2(\alpha)} \int_a^b \int_a^b (h(b) - h(s))^{\alpha-1} (h(b) - h(t))^{\alpha-1} h'(s)\, p(s) \\ & \times h'(t)\, q(t) \left[ (f(s) - f(t)) (g(s) - g(t)) \right] ds dt. \end{aligned}$$

Therefore,

$$\begin{aligned}
& \left[I_{a,h}^{\alpha}p(b)\right]\left[I_{a,h}^{\alpha}\left(qfg\right)(b)\right]+\left[I_{a,h}^{\alpha}q(b)\right]\left[I_{a,h}^{\alpha}\left(pfg\right)(b)\right] \\
& -\left[I_{a,h}^{\alpha}\left(pf\right)(b)\right]\left[I_{a,h}^{\alpha}\left(qg\right)(b)\right]-\left[I_{a,h}^{\alpha}\left(qf\right)(b)\right]\left[I_{a,h}^{\alpha}\left(pg\right)(b)\right] \quad (3.2)\\
= & \frac{1}{\Gamma^{2}(\alpha)}\int_{a}^{b}\int_{a}^{b}(h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s) \\
& \times h'(t)\,q(t)\,(f(s)-f(t))\left(\int_{t}^{s}g'(x)dx\right)dsdt.
\end{aligned}$$

Using the condition $a\leq t\leq x\leq s\leq b$, we observe that

$$\begin{aligned}
& \left[I_{a,h}^{\alpha}p(b)\right]\left[I_{a,h}^{\alpha}\left(qfg\right)(b)\right]+\left[I_{a,h}^{\alpha}q(b)\right]\left[I_{a,h}^{\alpha}\left(pfg\right)(b)\right] \\
& -\left[I_{a,h}^{\alpha}\left(pf\right)(b)\right]\left[I_{a,h}^{\alpha}\left(qg\right)(b)\right]-\left[I_{a,h}^{\alpha}\left(qf\right)(b)\right]\left[I_{a,h}^{\alpha}\left(pg\right)(b)\right] \\
= & \frac{1}{\Gamma^{2}(\alpha)}\int_{a}^{b}\left[\int_{a}^{x}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)\right. \\
& \left.\times\int_{a}^{b}(h(b)-h(s))^{\alpha-1}(f(s)-f(t))\,h'(s)\,p(s)dsdt\right]\left(g'(x)\right)dx.
\end{aligned}$$

The particular case when $f(x)=x$ allows us to write

$$\begin{aligned}
& \left[I_{a,h}^{\alpha}p(b)\right]\left[I_{a,h}^{\alpha}b\left(qg\right)(b)\right]+\left[I_{a,h}^{\alpha}q(b)\right]\left[I_{a,h}^{\alpha}b\left(pg\right)(b)\right] \\
& -\left[I_{a,h}^{\alpha}bp(b)\right]\left[I_{a,h}^{\alpha}\left(qg\right)(b)\right]-\left[I_{a,h}^{\alpha}bq(b)\right]\left[I_{a,h}^{\alpha}\left(pg\right)(b)\right] \\
= & \frac{1}{\Gamma^{2}(\alpha)}\int_{a}^{b}\left[\int_{a}^{x}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)\right. \\
& \left.\times\int_{a}^{b}(h(b)-h(s))^{\alpha-1}(s-t)h'(s)\,p(s)dsdt\right]\left(g'(x)\right)dx \\
= & \frac{1}{\Gamma(\alpha)}\int_{a}^{b}\left[\frac{1}{\Gamma(\alpha)}\int_{a}^{b}(h(b)-h(s))^{\alpha-1}h'(s)\,sp(s)ds\right. \\
& \times\int_{a}^{x}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt \\
& -\frac{1}{\Gamma(\alpha)}\int_{a}^{b}(h(b)-h(s))^{\alpha-1}h'(s)\,p(s)ds \quad (3.3)\\
& \left.\times\int_{a}^{x}(h(b)-h(t))^{\alpha-1}h'(t)\,tq(t)dt\right]\left(g'(x)\right)dx.
\end{aligned}$$

To conclude, we use (3.2) and (3.3). □

Based on the above lemma, we prove the following theorem.

**Theorem 3.2.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$*

*on $(a,b)$, $\varphi : [a,b] \to \mathbb{R}$ is an absolutely continuous function and $p, q : [a,b] \to \mathbb{R}^+$ are positive integrable functions. If $(\varphi')^2 \in L^1[a,b]$, then for all $\alpha > 0$, we have*

$$\begin{aligned}
&\left[I^{\alpha}_{a,h}p(b)\right]\left[I^{\alpha}_{a,h}\left(q\varphi^2\right)(b)\right] + \left[I^{\alpha}_{a,h}q(b)\right]\left[I^{\alpha}_{a,h}\left(p\varphi^2\right)(b)\right]\\
&-2\left[I^{\alpha}_{a,h}\left(p\varphi\right)(b)\right]\left[I^{\alpha}_{a,h}\left(q\varphi\right)(b)\right] \qquad (3.4)\\
\leq\ &\frac{1}{\Gamma(\alpha)}\int_a^b\left(I^{\alpha}_{a,h}bp(b)\int_a^x (h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right.\\
&\left.-I^{\alpha}_{a,h}p(b)\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right)\left(\varphi'(x)\right)^2 dx.
\end{aligned}$$

*Proof.* Using Definition 2.2 and Lemma 3.1, we have

$$\begin{aligned}
&I^{\alpha}_{a,h}q(b)I^{\alpha}_{a,h}\left(p\varphi^2\right)(b) + I^{\alpha}_{a,h}p(b)I^{\alpha}_{a,h}\left(q\varphi^2\right)(b) - 2I^{\alpha}_{a,h}\left(p\varphi\right)(b)I^{\alpha}_{a,h}\left(q\varphi\right)(b)\\
&= \tfrac{1}{\Gamma^2(\alpha)}\int_a^b\int_a^b (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\\
&\qquad \times h'(t)\,q(t)\left[\varphi(s)-\varphi(t)\right]^2 dsdt\\
&= \tfrac{1}{\Gamma^2(\alpha)}\int_a^b\int_a^b (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\\
&\qquad \times h'(t)\,q(t)(s-t)^2\left(\tfrac{\varphi(s)-\varphi(t)}{s-t}\right)^2 dsdt.
\end{aligned}$$

Hence,

$$\begin{aligned}
&I^{\alpha}_{a,h}q(b)I^{\alpha}_{a,h}\left(p\varphi^2\right)(b) + I^{\alpha}_{a,h}p(b)I^{\alpha}_{a,h}\left(q\varphi^2\right)(b) - 2I^{\alpha}_{a,h}\left(p\varphi\right)(b)I^{\alpha}_{a,h}\left(q\varphi\right)(b)\\
&= \tfrac{1}{\Gamma^2(\alpha)}\int_a^b\int_a^b (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\\
&\qquad \times h'(t)\,q(t)(s-t)^2\left[\tfrac{\int_t^s(\varphi'(x))dx}{s-t}\right]^2 dsdt.
\end{aligned}$$

Thanks to Cauchy-Schwarz inequality (see [5]), it yields that

$$\begin{aligned}
&I^{\alpha}_{a,h}q(b)I^{\alpha}_{a,h}\left(p\varphi^2\right)(b) + I^{\alpha}_{a,h}p(b)I^{\alpha}_{a,h}\left(q\varphi^2\right)(b) - 2I^{\alpha}_{a,h}\left(p\varphi\right)(b)I^{\alpha}_{a,h}\left(q\varphi\right)(b)\\
&= \tfrac{1}{\Gamma^2(\alpha)}\int_a^b\int_a^b (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\\
&\qquad \times h'(t)\,q(t)(s-t)^2\left(\tfrac{\int_t^s\varphi'(x)dx}{s-t}\right)^2 dsdt \qquad (3.5)\\
&\leq \tfrac{1}{\Gamma^2(\alpha)}\int_a^b\int_a^b (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\\
&\qquad \times h'(t)\,q(t)(s-t)^2\left(\frac{\left(\int_t^s dx\right)^{\frac12}\left(\int_t^s(\varphi'(x))^2dx\right)^{\frac12}}{s-t}\right)^2 dsdt\\
&\leq \tfrac{1}{\Gamma^2(\alpha)}\int_a^b\int_a^b (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\\
&\qquad \times h'(t)\,q(t)(s-t)\left(\int_t^s\left(\varphi'(x)\right)^2dx\right)dsdt.
\end{aligned}$$

Finally, using (3.1) and (3.5), we get (3.4). □

**Corollary 3.3.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$, $\varphi : [a,b] \to \mathbb{R}$ is an absolutely continuous function and $q$ is a positive integrable function on $[a,b]$. If $(\varphi')^2 \in L^1[a,b]$, then for all $\alpha > 0$, we have*

$$\begin{aligned} & \frac{(h(b)-h(a))^\alpha}{\Gamma(\alpha+1)} I_{a,h}^\alpha \left(q\varphi^2\right)(b) + \left[I_{a,h}^\alpha q(b)\right]\left[I_{a,h}^\alpha \left(\varphi^2\right)(b)\right] \\ & -2\left[I_{a,h}^\alpha (\varphi)(b)\right]\left[I_{a,h}^\alpha (q\varphi)(b)\right] \\ \leq\ & \frac{1}{\Gamma(\alpha)} \int_a^b \left[ I_{a,h}^\alpha b \int_a^x (h(b)-h(t))^{\alpha-1} h'(t)\, q(t)\, dt - \frac{(h(b)-h(a))^\alpha}{\Gamma(\alpha+1)} \right. \\ & \left. \times \int_a^x t(h(b)-h(t))^{\alpha-1} h'(t)\, q(t)\, dt \right] \left(\varphi'(x)\right)^2 dx. \end{aligned}$$

*Proof.* We apply Theorem 3.2 with $p(x) = 1, x \in [a,b]$. □

**Corollary 3.4.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$, $\varphi : [a,b] \to \mathbb{R}$ is an absolutely continuous function. If $(\varphi')^2 \in L^1[a,b]$ and $p$ is a positive integrable function on $[a,b]$, then for all $\alpha > 0$, we have*

$$\begin{aligned} I_{a,h}^\alpha p(b) I_{a,h}^\alpha \left(\varphi^2\right)(b) + \tfrac{(h(b)-h(a))^\alpha}{\Gamma(\alpha+1)} I_{a,h}^\alpha \left(p\varphi^2\right)(b) - 2\Big[I_{a,h}^\alpha (p\varphi)(b)\Big]\Big[I_{a,h}^\alpha (\varphi)(b)\Big] \\ \leq \tfrac{1}{\Gamma(\alpha)} \textstyle\int_a^b \left[ I_a^\alpha b p(b) \int_a^x (h(b)-h(t))^{\alpha-1} h'(t)\, dt \right. \\ \left. - I_a^\alpha p(b) \textstyle\int_a^x t(h(b)-h(t))^{\alpha-1} h'(t)\, dt \right] \left(\varphi'(x)\right)^2 dx. \end{aligned}$$

*Proof.* In Theorem 3.2, we take $q(x) = 1, x \in [a,b]$. □

**Corollary 3.5.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$, $\varphi : [a,b] \to \mathbb{R}$ is an absolutely continuous function. If $(\varphi')^2 \in L^1[a,b]$, then for all $\alpha > 0$, we have*

$$\begin{aligned} & \frac{(h(b)-h(a))^\alpha}{\Gamma(\alpha+1)} \left[I_{a,h}^\alpha \left(\varphi^2\right)(b)\right] - \left[I_{a,h}^\alpha (\varphi)(b)\right]^2 \qquad (3.6) \\ \leq\ & \frac{1}{2\Gamma(\alpha)} \int_a^b \left( I_{a,h}^\alpha b \int_a^x (h(b)-h(t))^{\alpha-1} h'(t)\, dt - \frac{(h(b)-h(a))^\alpha}{\Gamma(\alpha+1)} \right. \\ & \left. \times \int_a^x t(h(b)-h(t))^{\alpha-1} h'(t)\, dt \right) \left(\varphi'(x)\right)^2 dx. \end{aligned}$$

*Proof.* By Theorem 3.2, we take $p(x) = q(x) = 1, x \in [a,b]$ in (3.4), we deduce (3.6). □

Another main result that needs to be proved is the following theorem:

**Theorem 3.6.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $f,g:[a,b]\to\mathbb{R}$ are two absolutely continuous functions on $[a,b]$, $p,q:[a,b]\to\mathbb{R}^+$ are integrable. If $(f')^2,(g')^2\in L^1[a,b]$, then for any $\alpha>0$, we have*

$$\begin{aligned}
&\big|I^{\alpha}_{a,h}p(b)I^{\alpha}_{a,h}qfg(b)+I^{\alpha}_{a,h}q(b)I^{\alpha}_{a,h}pfg(b)\\
&-\left(I^{\alpha}_{a,h}pf(b)\right)\left(I^{\alpha}_{a,h}qg(b)\right)-\left(I^{\alpha}_{a,h}qf(b)\right)\left(I^{\alpha}_{a,h}pg(b)\right)\big|\\
\leq\ &\frac{1}{\Gamma(\alpha)}\left(\int_a^b\left[I^{\alpha}_{a,h}bp(b)\int_a^x(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right.\right.\\
&\left.\left.-I^{\alpha}_{a,h}p(b)\int_a^x t\,(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right]\left(f'(x)\right)^2dx\right)^{\frac{1}{2}}\\
&\times\left(\int_a^b\left[I^{\alpha}_{a,h}bp(b)\int_a^x(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right.\right.\\
&\left.\left.-I^{\alpha}_{a,h}p(b)\int_a^x t\,(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right]\left(g'(x)\right)^2dx\right)^{\frac{1}{2}}.
\end{aligned}$$

*Proof.* We can prove that

$$\begin{aligned}
&\big|I^{\alpha}_{a,h}p(b)I^{\alpha}_{a,h}(qfg)(b)+I^{\alpha}_{a,h}q(b)I^{\alpha}_{a,h}(pfg)(b)\\
&-I^{\alpha}_{a,h}(pf)(b)I^{\alpha}_{a,h}(qg)(b)\left(I^{\alpha}_{a,h}qf(b)\right)\left(I^{\alpha}_{a,h}pg(b)\right)\big|\\
\leq\ &\frac{1}{\Gamma^2(\alpha)}\left(\int_a^b\int_a^b(h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\right.\\
&\left.\times h'(t)\,q(t)\,(f(s)-f(t))^2\,dsdt\right)^{\frac{1}{2}}\\
&\times\left(\int_a^b\int_a^b(h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\right.\\
&\left.\times h'(t)\,q(t)\,(g(s)-g(t))^2\,dsdt\right)^{\frac{1}{2}}\\
\leq\ &\frac{1}{\Gamma^2(\alpha)}\left(\int_a^b\int_a^b(h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\right.\\
&\left.\times h'(t)\,q(t)\left(\int_t^s f'(x)dx\right)^2dsdt\right)^{\frac{1}{2}}\\
&\times\left(\int_a^b\int_a^b(h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)\right.\\
&\left.\times h'(t)\,q(t)\left(\int_t^s g'(x)dx\right)^2dsdt\right)^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\leq\ & \frac{1}{\Gamma^2(\alpha)}\left[\int_a^b\left(\int_a^b s(h(b)-h(s))^{\alpha-1}h'(s)\,p(s)ds\right.\right.\\
&\times\int_a^x (h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt-\int_a^b (h(b)-h(s))^{\alpha-1}h'(s)\,p(s)ds\\
&\left.\left.\times\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right)\left(f'(x)\right)^2dx\right]^{\frac{1}{2}}\\
&\times\left[\int_a^b\left(\int_a^b s(h(b)-h(s))^{\alpha-1}h'(s)\,p(s)ds\right.\right.\\
&\times\int_a^x (h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt-\int_a^b (h(b)-h(s))^{\alpha-1}h'(s)\,p(s)ds\\
&\left.\left.\times\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right)\left(g'(x)\right)^2dx\right]^{\frac{1}{2}}
\end{aligned}$$

Theorem 3.6 is thus proved. □

**Corollary 3.7.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $f,g:[a,b]\to\mathbb{R}$ are two absolutely continuous functions. If $(f')^2,(g')^2\in L^1[a,b]$ and $q$ is a positive integrable function on $[a,b]$, then for any $\alpha>0$, we have*

$$\begin{aligned}
&\left|\frac{(h(b)-h(a))^\alpha}{\Gamma(\alpha+1)}I^\alpha_{a,h}qfg(b)+I^\alpha_{a,h}q(b)I^\alpha_{a,h}fg(b)\right.\\
&\left.-\left(I^\alpha_{a,h}f(b)\right)\left(I^\alpha_{a,h}qg(b)\right)-\left(I^\alpha_{a,h}qf(b)\right)\left(I^\alpha_{a,h}g(b)\right)\right|\\
\leq\ & \frac{1}{\Gamma(\alpha)}\left(\int_a^b\left[I^\alpha_{a,h}b\int_a^x (h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right.\right.\\
&\left.\left.-\frac{(h(b)-h(a))^\alpha}{\Gamma(\alpha+1)}\int_a^x t\,(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right]\left(f'(x)\right)^2dx\right)^{\frac{1}{2}}\\
&\times\left(\int_a^b\left[I^\alpha_{a,h}b\int_a^x (h(b)-h(t))^{\alpha-1}h'(s)\,q(t)dt\right.\right.\\
&\left.\left.-\frac{(h(b)-h(a))^\alpha}{\Gamma(\alpha+1)}\int_a^x t\,(h(b)-h(t))^{\alpha-1}h'(s)\,q(t)dt\right]\left(g'(x)\right)^2dx\right)^{\frac{1}{2}}.
\end{aligned}$$

*Proof.* We apply Theorem 3.6 with $p(x)=1, x\in[a,b]$. □

**Corollary 3.8.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$*

*on $(a,b)$ and $f,g:[a,b]\to\mathbb{R}$ are two absolutely continuous functions. If $(f')^2,(g')^2\in L^1[a,b]$, and $p$ is a positive integrable function on $[a,b]$, then for any $\alpha>0$, we have*

$$\begin{aligned}&\Big|I^{\alpha}_{a,h}p(b)I^{\alpha}_{a,h}fg(b)+\frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)}I^{\alpha}_{a,h}pfg(b)\\&-\left(I^{\alpha}_{a,h}pf(b)\right)\left(I^{\alpha}_{a,h}g(b)\right)-\left(I^{\alpha}_{a,h}f(b)\right)\left(I^{\alpha}_{a,h}pg(b)\right)\Big|\\ \leq\ &\frac{1}{\Gamma(\alpha)}\left(\int_a^b\left[I^{\alpha}_{a,h}bp(b)\int_a^x(h(b)-h(t))^{\alpha-1}h'(t)\,dt\right.\right.\\&\left.\left.-I^{\alpha}_{a,h}p(b)\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,dt\right]\left(f'(x)\right)^2dx\right)^{\frac12}\\&\times\left(\int_a^b\left[I^{\alpha}_{a,h}bp(b)\int_a^x(h(b)-h(t))^{\alpha-1}h'(t)\,dt\right.\right.\\&\left.\left.-I^{\alpha}_{a,h}p(b)\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,dt\right]\left(g'(x)\right)^2dx\right)^{\frac12}.\end{aligned}$$

*Proof.* In Theorem 3.6, we take $q(x)=1, x\in[a,b]$. □

**Corollary 3.9.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $f,g:[a,b]\to\mathbb{R}$ are two absolutely continuous functions. If $(f')^2,(g')^2\in L^1[a,b]$, then for any $\alpha>0$, we have*

$$\begin{aligned}&\left|\frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)}I^{\alpha}_{a,h}fg(b)-\left(I^{\alpha}_{a,h}f(b)\right)\left(I^{\alpha}_{a,h}g(b)\right)\right|\\ \leq\ &\frac{1}{2\Gamma(\alpha)}\left(\int_a^b\left[I^{\alpha}_{a,h}b\int_a^x(h(b)-h(t))^{\alpha-1}h'(t)\,dt\right.\right. \qquad (3.7)\\&\left.\left.-\frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)}\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,dt\right]\left(f'(x)\right)^2dx\right)^{\frac12}\\&\times\left(\int_a^b\left[I^{\alpha}_{a,h}b\int_a^x(h(b)-h(t))^{\alpha-1}h'(t)\,dt\right.\right.\\&\left.\left.-\frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)}\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,dt\right]\left(g'(x)\right)^2dx\right)^{\frac12}.\end{aligned}$$

*Proof.* We take $p(x)=q(x)=1, x\in[a,b]$ in Theorem 3.6 we get (3.7). □

In the case of one nondecreasing function, we prove the following result:

**Theorem 3.10.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $g$ be a nondecreasing function on $[a,b]$, $f$ be an absolutely* *continuous function on $[a,b]$ and suppose that $p,q$ are positive integrable functions on $[a,b]$ . If $f' \in L^{\infty}[a,b]$, then for any $\alpha > 0$, we have*

$$
\begin{aligned}
&\left|I_{a,h}^{\alpha}p(b)I_{a,h}^{\alpha}qfg(b)+I_{a,h}^{\alpha}q(b)I_{a,h}^{\alpha}pfg(b)\right.\\
&\left.-I_{a,h}^{\alpha}pf(b)I_{a,h}^{\alpha}qg(b)-I_{a,h}^{\alpha}qf(b)I_{a,h}^{\alpha}pg(b)\right| \qquad (3.8)\\
\leq\; & \frac{||f'||_{\infty}}{\Gamma(\alpha)}\left[I_{a,h}^{\alpha}bp(b)\int_a^x (h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt - I_{a,h}^{\alpha}p(b)\right.\\
&\left.\times\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right]\int_a^b g'(x)dx.
\end{aligned}
$$

*Proof.* We have

$$
\begin{aligned}
&\left|I_{a,h}^{\alpha}p(b)I_{a,h}^{\alpha}qfg(b)+I_{a,h}^{\alpha}q(b)I_{a,h}^{\alpha}pfg(b)\right.\\
&\left.-I_{a,h}^{\alpha}pf(b)I_{a,h}^{\alpha}qg(b)-I_{a,h}^{\alpha}qf(b)I_{a,h}^{\alpha}pg(b)\right|\\
=\; & \frac{1}{\Gamma^2(\alpha)}\left|\int_a^b\int_a^b (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)h'(t)\,q(t)\right.\\
&\left.\times\left[(f(s)-f(t))\,(g(s)-g(t))\right]dsdt\right|\\
\leq\; & \frac{1}{\Gamma^2(\alpha)}\int_a^b\int_a^b (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)h'(t)\,q(t)\\
&\times\left|\frac{(f(s)-f(t))}{s-t}\right|\left|(s-t)\,(g(s)-g(t))\right|dsdt\\
\leq\; & \frac{||f'||_{\infty}}{\Gamma^2(\alpha)}\int_a^b\int_a^x (h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)h'(t)\,q(t)\\
&\times(s-t)\left(\int_a^b g'(x)dx\right)dsdt\\
\leq\; & \frac{||f'||_{\infty}}{\Gamma(\alpha)}\left[I_{a,h}^{\alpha}bp(b)\int_a^x (h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt - I_{a,h}^{\alpha}p(b)\right.\\
&\left.\times\int_a^x t(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right]\int_a^b g'(x)dx.
\end{aligned}
$$

Theorem 3.10 is thus proved. □

**Corollary 3.11.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $g$ be a nondecreasing function on $[a,b]$, $f$ be an absolutely continuous function on $[a,b]$. If $f' \in L^{\infty}[a,b]$ and $q$ is a positive integrable function on $[a,b]$, then for any $\alpha > 0$, we have*

$$\begin{aligned}
&\left|\frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)} I_{a,h}^{\alpha} qfg(b) + I_{a,h}^{\alpha} q(b) I_{a,h}^{\alpha} fg(b)\right. \\
&\left. - I_{a,h}^{\alpha} f(b) I_{a,h}^{\alpha} qg(b) - I_{a,h}^{\alpha} qf(b) I_{a,h}^{\alpha} g(b)\right| \qquad (3.9) \\
\leq\ & \frac{\|f'\|_{\infty}}{\Gamma(\alpha)} \left[ I_{a,h}^{\alpha} b \int_a^x (h(b)-h(t))^{\alpha-1} h'(t)\, q(t) dt - \frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)} \right. \\
&\left. \times \int_a^x t (h(b)-h(t))^{\alpha-1} h'(t)\, q(t) dt \right] \int_a^b g'(x) dx.
\end{aligned}$$

*Proof.* We take $p(x) = 1, x \in [a,b]$ in (3.8), we deduce (3.9). □

**Corollary 3.12.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $g$ be a nondecreasing function on $[a,b]$, $f$ be an absolutely continuous function on $[a,b]$. If $f' \in L^{\infty}[a,b]$ and $p$ is a positive integrable function on $[a,b]$, then for any $\alpha > 0$, we have*

$$\begin{aligned}
&\left| I_{a,h}^{\alpha} p(b) I_{a,h}^{\alpha} fg(b) + \frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)} I_{a,h}^{\alpha} pfg(b) - 2 I_{a,h}^{\alpha} f(b) I_{a,h}^{\alpha} qg(b) \right| \\
\leq\ & \frac{\|f'\|_{\infty}}{\Gamma(\alpha)} \left[ I_{a,h}^{\alpha} b p(b) \int_a^x (h(b)-h(t))^{\alpha-1} h'(t)\, dt - I_{a,h}^{\alpha} p(b) \right. \qquad (3.10) \\
&\left. \times \int_a^x t (h(b)-h(t))^{\alpha-1} h'(t)\, dt \right] \int_a^b g'(x) dx.
\end{aligned}$$

*Proof.* We take $q(x) = 1, x \in [a,b]$ in (3.8), we deduce (3.10). □

**Corollary 3.13.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $g$ be a nondecreasing function on $[a,b]$, $f$ be an absolutely continuous function on $[a,b]$. If $f' \in L^{\infty}[a,b]$, then for any $\alpha > 0$, we have*

$$\begin{aligned}
&\left| \frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)} I_{a,h}^{\alpha} fg(b) - I_{a,h}^{\alpha} f(b) I_{a,h}^{\alpha} g(b) \right| \qquad (3.11) \\
\leq\ & \frac{\|f'\|_{\infty}}{2\Gamma(\alpha)} \left[ I_{a,h}^{\alpha} b \int_a^x (h(b)-h(t))^{\alpha-1} h'(t)\, dt - \frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)} \right. \\
&\left. \times \int_a^x t (h(b)-h(t))^{\alpha-1} h'(t)\, dt \right] \int_a^b g'(x) dx.
\end{aligned}$$

*Proof.* We take $p(x) = q(x) = 1, x \in [a,b]$ in (3.8), we deduce (3.11). □

The main result corresponding to the case of two monotonic functions is given by the following theorem.

**Theorem 3.14.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $f,g:[a,b]\to\mathbb{R}$ be two absolutely monotonic functions on $[a,b]$ and $g$ is non decreasing on $[a,b]$ . If $f',g'\in L^{\infty}[a,b]$, then for any positive integrable functions $p,q$ defined on $[a,b]$, we have*

$$\begin{aligned}
&\Big| I_{a,h}^{\alpha}p(b)I_{a,h}^{\alpha}qfg(b)+I_{a,h}^{\alpha}q(b)I_{a,h}^{\alpha}pfg(b)\\
&-I_{a,h}^{\alpha}pf(b)I_{a,h}^{\alpha}qg(b)-I_{a,h}^{\alpha}qf(b)I_{a,h}^{\alpha}pg(b)\Big| \qquad (3.12)\\
\leq\ &\frac{||f'||_{\infty}||g'||_{\infty}}{\Gamma(\alpha)}\Big[I_{a,h}^{\alpha}b^{2}p(b)\int_{a}^{x}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\\
&-2I_{a,h}^{\alpha}bp(b)\int_{a}^{x}t(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\\
&+I_{a,h}^{\alpha}p(b)\int_{a}^{x}t^{2}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\Big].
\end{aligned}$$

*Proof.* We have,

$$\begin{aligned}
&\Big| I_{a,h}^{\alpha}p(b)I_{a,h}^{\alpha}qfg(b)+I_{a,h}^{\alpha}q(b)I_{a,h}^{\alpha}pfg(b)\\
&-I_{a,h}^{\alpha}pf(b)I_{a,h}^{\alpha}qg(b)-I_{a,h}^{\alpha}qf(b)I_{a,h}^{\alpha}pg(b)\Big|\\
=\ &\frac{1}{\Gamma^{2}(\alpha)}\Big|\int_{a}^{b}\int_{a}^{b}(h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)h'(t)\,q(t)\\
&\times\left[(f(s)-f(t))\,(g(s)-g(t))\right]dsdt\Big|\\
\leq\ &\frac{1}{\Gamma^{2}(\alpha)}\int_{a}^{b}\int_{a}^{b}(h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}h'(s)\,p(s)h'(t)\,q(t)\\
&\times\left|\frac{f(s)-f(t)}{s-t}\right|\left|\frac{g(s)-g(t)}{s-t}\right|(s-t)^{2}dsdt\\
\leq\ &\frac{||f'||_{\infty}||g'||_{\infty}}{\Gamma^{2}(\alpha)}\int_{a}^{b}\int_{a}^{x}(h(b)-h(s))^{\alpha-1}(h(b)-h(t))^{\alpha-1}\\
&\times h'(s)\,p(s)h'(t)\,q(t)(s-t)^{2}dsdt\\
\leq\ &\frac{||f'||_{\infty}||g'||_{\infty}}{\Gamma(\alpha)}\Big[I_{a,h}^{\alpha}b^{2}p(b)\int_{a}^{x}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\\
&-2I_{a,h}^{\alpha}bp(b)\int_{a}^{x}t(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\\
&+I_{a}^{\alpha}p(b)\int_{a}^{x}t^{2}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\Big].
\end{aligned}$$

Theorem 3.14 is thus proved. □

**Corollary 3.15.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $f,g:[a,b]\to\mathbb{R}$ be two absolutely monotonic functions on $[a,b]$ and $g$ be a non decreasing on $[a,b]$. If $f',g'\in L^{\infty}[a,b]$ and $q$ is a positive integrable function on $[a,b]$, then for $\alpha>0$, we have the inequality*

$$\begin{aligned}
&\left|\frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)}I^{\alpha}_{a,h}qfg(b)+I^{\alpha}_{a,h}q(b)I^{\alpha}_{a,h}fg(b)\right.\\
&\left.-I^{\alpha}_{a,h}f(b)I^{\alpha}_{a,h}qg(b)-I^{\alpha}_{a,h}qf(b)I^{\alpha}_{a,h}g(b)\right| \qquad (3.13)\\
\leq\quad&\frac{||f'||_{\infty}||g'||_{\infty}}{\Gamma(\alpha)}\left[I^{\alpha}_{a,h}b^{2}\int_{a}^{x}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right.\\
&-2I^{\alpha}_{a,h}b\int_{a}^{x}t(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\\
&\left.+\frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)}\int_{a}^{x}t^{2}(h(b)-h(t))^{\alpha-1}h'(t)\,q(t)dt\right].
\end{aligned}$$

*Proof.* We take $p(x)=1, x\in[a,b]$ in (3.12), we deduce (3.13). □

**Corollary 3.16.** *Assume that $h$ is a measurable, increasing, positive and monotone function on $(a,b]$ and $h(t)$ having a continuous derivative $h'(t)$ on $(a,b)$ and $f,g:[a,b]\to\mathbb{R}$ be two absolutely monotonic functions on $[a,b]$ and $g$ be a non decreasing on $[a,b]$. If $f',g'\in L^{\infty}[a,b]$ and $p$ is a positive integrable function, then for $\alpha>0$, we have*

$$\begin{aligned}
&\left|I^{\alpha}_{a,h}p(b)I^{\alpha}_{a,h}fg(b)+\frac{(h(b)-h(a))^{\alpha}}{\Gamma(\alpha+1)}I^{\alpha}_{a,h}pfg(b)\right.\\
&\left.-I^{\alpha}_{a,h}pf(b)I^{\alpha}_{a,h}g(b)-I^{\alpha}_{a,h}f(b)I^{\alpha}_{a,h}pg(b)\right|\\
\leq\quad&\frac{||f'||_{\infty}||g'||_{\infty}}{\Gamma(\alpha)}\left[I^{\alpha}_{a,h}b^{2}p(b)\int_{a}^{x}(h(b)-h(t))^{\alpha-1}h'(t)\,dt-2I^{\alpha}_{a,h}bp(b)\right.\\
&\left.\times\int_{a}^{x}\left[t(h(b)-h(t))^{\alpha-1}h'(t)+I^{\alpha}_{a,h}p(b)t^{2}(h(b)-h(t))^{\alpha-1}h'(t)\right]dt\right].
\end{aligned}$$

*Proof.* It just take $q(x)=1, x\in[a,b]$ in the Theorem 3.14. □

*Remark* 3.17. :

1) Taking $p(x)=q(x)$ and $h(x)=x$ in Theorem 3.2, we obtain Theorem 3.1 of [2].
2) Taking $p(x)=q(x)=1$ and $h(x)=x$ in Corollary 3.3 and Corollary 3.4, we obtain Corollary 3.1 of [2].
3) Taking $p(x)=q(x)$ and $h(x)=x$ in Theorem 3.6, we obtain Theorem 3.2 of [2].

4) Taking $p(x) = q(x)$ and $h(x) = x$ in Theorem 3.10, we obtain Theorem 3.3 of [2].
5) Taking $p(x) = q(x)$ and $h(x) = x$ in Theorem 3.14, we obtain Theorem 3.4 of [2].

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Mohamed BEZZIOU
University UDBKM and Laboratory LPAM,
FSEI, University of Mostaganem, Algeria
Department of Mathematics
e-mail: m.bezziou@univ-dbkm.dz

Zoubir DAHMANI and Amina KHAMELI
Laboratory LPAM, FSEI,
University of Mostaganem, Algeria
e-mail: zzdahmani@yahoo.fr
e-mail: khameli.amina@gmail.com