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A NOTE ON INVERSE SYSTEMS OF $S(n)$-CLOSED SPACES

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Abstract. The aim of this paper is to study inverse systems of the $S(n)$-closed spaces which are the generalization of H-closed and Urysohn-closed spaces.

1. Introduction

In this paper the symbol $\mathbb{N}^+$ denotes the set of positive integers and $\mathbb{N} = (0) \cup \mathbb{N}^+$.

The concept of *θ-closure* was introduced by Veličko [16]. For a subset M of a topological space X the *θ-closure* is defined by $Cl_\theta M = \{x \in X$: every closed neighborhood of x meets $M\}$, M is *θ-closed* if $Cl_\theta M = M$. This concept was used by many authors for the study of Hausdorff non-regular spaces. The θ-closure is related especially to Urysohn spaces (every pair of distinct points can be separated by disjoint closed neighborhoods). A space X is Urysohn iff the diagonal in $X \times X$ is θ-closed.

We say that a pair (G, H) is an *ordered pair* of open sets about $x \in X$ if G and H are open subsets of X and $x \in G \subset ClG \subset H$. A point $x \in X$ is in the *u-closure* of a subset $K \subset X$ ($x \in Cl_u K$) if each ordered pair (G, H) of open sets about $x \in X$ satisfies $K \cap ClH \neq \emptyset$. A subset K of a space X is *u-closed* if $K = Cl_u K$.

A generalization of the concepts of θ-closure and of u-closure *is* θ^n-*closure* defined in Section 2. Section 3 is the main section of this paper. In this section we study the inverse systems of S(n)-closed spaces. We shall show that an inverse limit of S(n)-closed space and Θ^n-closed bonding mapping p_{ab} is non-empty (Theorem 3.5). Moreover, if the projections $p_a : \lim \mathbf{X} \to \mathbf{X}_a, a \in A$, are Θ^n-closed and the θ^n-*closure* $Cl_{\theta^n} M$ is *Kuratowski Closure Operator i.e.* $Cl_{\theta^n}(Cl_{\theta^n}(A)) = Cl_{\theta^n}(A)$, then $X = \lim \mathbf{X}$ is non-empty and S(n)-closed (Theorem 3.7).

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2. S(n)-spaces

For a positive integer n and a subset M of a topological space X, the θ^n-*closure* $Cl_{\theta^n}M$ of M is defined to be the set [3]

$$\begin{aligned} \{x &\in X : \text{for every chain of open neighborhoods of } x, \\ \text{if } U_1 &\subset U_2 \subset \cdots \subset U_n \text{ with } Cl(U_i) \subset U_{i+1}, \\ \text{where } i &= 1, 2, \ldots, n-1, \text{ then one has } Cl(U_n) \cap M \neq \emptyset\}. \end{aligned}$$

For $n = 1$ this gives θ-closure. Moreover, for $n = 2$ the above definition gives u-closure.

Definition 2.1. A subset M of X is said to be θ^n-*closed* if $M = Cl_{\theta^n}M$. Similarly θ^n-*interior* of M is defined and denoted by $\text{Int}_{\theta^n}M$, so $\text{Int}_{\theta^n}M = X \diagdown Cl_{\theta^n}(X \backslash M)$.

Proposition 1. *Every θ^n- closed subset $M \subset X$ is closed.*

Proof. See [14, p. 222]. □

Definition 2.2. An open set U is called a *n-hull* of a set A (see [9, p. 624]) if there exists a family of open sets $U_1, U_2, \ldots, U_n = U$ such that $A \subset U_1$ and $ClU_i \subset U_{i+1}$ for $i = 1, \ldots, n-1$.

Definition 2.3. For $n \in \mathbb{N}$ and a filter $\mathcal{F}$ on X we denote by $\text{ad}_{\theta^n}\mathcal{F}$ the set of θ^n- adherent points of $\mathcal{F}$, i.e. $\text{ad}_{\theta^n}\mathcal{F} = \cap\{Cl_{\theta^n}F_\alpha : F_\alpha \in \mathcal{F}\}$. In particular $\text{ad}_{\theta^0}\mathcal{F} = \text{ad}\mathcal{F}$ is the set of adherent points of $\mathcal{F}$.

Definition 2.4. Let X be a space and $n \in \mathbb{N}$; a point x of X is $S(n)$-*separated from a subset* M of X if $x \notin Cl_{\ \theta^n}M$. In particular x is $S(0)$-separated from M if $x \notin ClM$.

Definition 2.5. Let $n \in \mathbb{N}$ and X be a *space*:

(a) X is an $S(n)$-*space* if every pair of distinct points of X are S(n)-separated;

(b) A filter $\mathcal{F}$ on X is an $S(n)$-*filter* if every nonadherent point of $\mathcal{F}$ is $S(n)$-separated from some member of $\mathcal{F}$;

(c) An open cover $\{U_\alpha\}$ of X is an $S(n)$-*cover* if every point of X is in the θ^n-interior of some U_α.

The $S(n)$-spaces coincide with the $\overline{T}_n$-spaces defined in [17] and studied further in [10], where also $S(\alpha)$-spaces are defined for each ordinal α.

Proposition 2. *The $S(0)$-spaces are the T_0 spaces, the $S(1)$-spaces are the Hausdorff spaces and the $S(2)$-spaces are the Urysohn spaces.*

Clearly every *filter* is an $S(0)$-*filter,* every open cover is an $S(0)$-cover and every open filter is an $S(1)$-filter. The open $S(2)$-filters coincide with the

Urysohn filters defined in [6] and [12]. For $n \geq 1$ the open S(n)-filters were defined in [10]. The special covers used in (3.9) [10] are $S(n-1)$ covers, $S(2)$-covers are the Urysohn covers *defined* in [1]. In a regular space every filter (resp. open cover) is an S(n)-filter (resp. $S(n)$-cover) for every $n \in \mathbb{N}$.

The following proposition plays fundamental role in this paper.

Proposition 3. *In any topological space:*

a) *The empty set and the whole space are Θ^n-closed,*
b) *An arbitrary finite union of Θ^n-closed sets is Θ^n-closed,*
c) *An arbitrary intersection of Θ^n-closed sets is Θ^n-closed,*
d) *A Θ^n-closed subset is closed,*
e) *$ClK \subset Cl_{\Theta^n}K$ for each subset K.*

Proof. **a)** By definition.

b) Let $F = \cup\{F_i : i = 1, \dots, n\}$ where each F_i is Θ^n-closed. For each $x \notin F$ there exist n-hull U_i of x such that $ClU_i \cap F_i = \emptyset$, $i = 1 \dots, n$. Now $U = \cap\{U_i : i = 1, \dots, n\}$ is n-hull of x such that $ClU \cap F = \emptyset$. This means that $x \notin Cl_{\theta^n}F$, i.e. F is Θ^n-closed.

c) Assume that $x \in Cl_{\theta^n}F$, where $F = \cap\{F_\alpha : \alpha \in A\}$ and each F_α is θ^n- closed. This means that for each n-hull U of the point x we have $ClU \cap F \neq \emptyset$. Clearly $ClU \cap F_\alpha \neq \emptyset$ for every $\alpha \in A$. We infer that $x \in F_\alpha$, $\alpha \in A$, since each F_α is θ^n- closed. Finally, $x \in \cap\{F_\alpha : \alpha \in A\} = F$ and F is θ^n- closed ($F = Cl_{\theta^n}F$).

d) See Proposition 1.

e) The set ClK is the minimal closed set containing K. Hence, $ClK \subset Cl_{\Theta^n}K$. □

From (a) and (b) we get the following.

Lemma 2.1. *If X is a topological space, then for each $Y \subset X$ there exists a minimal Θ^n-closed subset $Z \subset X$ such that $Y \subset Z$.*

Proof. The collection Φ of all Θ^n-closed subsets W of X which contains Y is non-empty since $X \in \Phi$. By (b) of Proposition 3 we infer that $Z = \cap\{W : W \in \Phi\}$ is a minimal Θ^n-closed subset $Z \subset X$ containing Y. □

Proposition 4. *If $n \geq 1$, then every θ^n- closed subset $M \subset X$ is $\theta-$ closed.*

Proof. If $n = 1$, then θ^n-closure gives the θ-closure. Suppose that $n \geq 2$ and that some θ^n- *closed subset* $M \subset X$ is not $\theta-$ closed. This means there exists a point $x \in X \setminus M$ such that for every open set U which contains x the set $ClU \cap M$ is non-empty. But x is not in θ^n- closure of M, thus there exists a chain of open neighborhoods of x, $U_1 \subset U_2 \subset \cdots \subset U_n$, such that $ClU_i \subset U_{i+1}$ for $i = 1, 2, \dots, n-1$ and $ClU_n \cap M = \emptyset$. This means that U_n is the neighborhood of x such that $ClU_n \cap M = \emptyset$. This contradicts the fact that for every open set U which contains x the set $ClU \cap M$ is non-empty. □

Definition 2.6. For a space $(X,\ \tau)$ and $n \in \mathbb{N}$ denote by $(X,\ \tau_{\theta^n})$, where τ_{θ^n} is the topology on X generated by the θ^n-closure, i.e. having as *closed* sets all θ^n-closed sets in $(X,\ \tau)$.

Clearly $\tau_{\theta^0} = \tau$, $\tau_{\theta^1} = \tau_\theta$ and a subset U of X is τ_{θ^n}-*open* iff every element of U is contained in the θ^n-interior of U.

The next proposition follows directly from the definitions.

Proposition 5. *For a topological space $(X,\ \tau)$ and $n \in \mathbb{N}^+$ the following conditions are equivalent:*

(a) *$(X,\ \tau)$ is an $S(n)$-space,*
(b) *$(X,\ \tau_{\theta^n})$ is a T_1 space*
(c) *$(X,\ \tau_{\theta^n})$ is T_0 and $(X,\ \tau)$ is T_1.*

If $n \geq 1$, then these conditions are equivalent to:
(d) *$(X,\ \tau_{\theta^n})$ is T_0.*

If $n = 2k$ with $k \in \mathbb{N}^+$, then the above conditions are equivalent to:
(e) *The diagonal in $X \times X$ is θ^k-closed*

This proposition show the pivotal rôle of the topologies τ_{θ^k} in the study of $S(n)$-spaces for $n > 1$. In some sense they replace the semiregularization which was the main tool in the study of *H-closed* spaces.

Let us observe that we have the following result.

Proposition 6. *The identity mapping $i : (X,\ \tau) \to (X,\ \tau_{\theta^n})$ is continuous.*

Definition 2.7. Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. We define a mapping $f_{\Theta^n} : (X, \tau_{\theta^n}) \to (Y, \sigma_{\theta^n})$ by $f_{\Theta^n}(x) = f(x)$ for every $x \in X$, i.e., the following diagram

$$\begin{array}{ccc} (X,\tau) & \xrightarrow{f} & (Y,\sigma) \\ \downarrow id & & \downarrow id \\ (X,\tau_{\theta^n}) & \xrightarrow{f_{\Theta^n}} & (Y,\sigma_{\theta^n}) \end{array} \tag{2.1}$$

commutes.

Lemma 2.2. *The mapping $f_{\Theta^n} : (X, \tau_{\theta^n}) \to (Y, \sigma_{\theta^n})$ is continuous.*

Proof. Let us prove that $f_{\Theta^n}^{-1}(F)$ is closed in (X, τ_{θ^n}) if F is closed in (Y, σ_{θ^n}). It suffices to prove that $f^{-1}(F)$ is Θ^n-closed in X if F is Θ^n-closed in Y. If $x \in X \setminus f^{-1}(F)$, then $f(x) \notin F$. There exists an open set U such that $f(x) \in U$ and $ClU \cap F = \emptyset$ since F is Θ^n-closed in Y. The open set $f^{-1}(U)$ contains x and $Clf^{-1}(U) \cap f^{-1}(F) = \emptyset$ since $f^{-1}(ClU) \cap f^{-1}(F) = \emptyset$. Hence, if $x \in X \setminus f^{-1}(F)$, then $x \in X \setminus Cl_{\Theta^n} f^{-1}(F)$, and, consequently, $f^{-1}(F)$ is Θ^n-closed in X. □

Proposition 7. *For every $M \subset X$ it follows*

$$cl_{\theta^n} M = \cap(ClU : U \text{ is n-hull of } M).$$

Proof. **a)** Let us prove that $cl_{\theta^n} M \subset \cap(ClU : U$ is n-hull of $M)$. If $x \in cl_{\theta^n} M$ then it is impossible that $x \notin \cap(ClU : U$ is n-hull of $M)$ since then there exists n-hull U of M such that $x \notin ClU$. This means that there exists a family of open sets $U_1, U_2, \ldots, U_n = U$ such that $A \subset U_1$ and $ClU_i \subset U_{i+1}$ for $i = 1, \ldots, n-1$. Now, $V_1 = X \setminus ClU_1 \supset V_2 = X \setminus ClU_2 \supset \cdots \supset V_n = X \setminus ClU_n$ is the open sets such that $x \in V_n \subset V_{n-1} \subset \cdots \subset V_1$ which satisfy the Definition 2.2. Hence, $x \notin cl_{\theta^n} M$, a contradiction.

b) Let us prove that

$$cl_{\theta^n} M \supset \cap(ClU : U \text{ is } n\text{-hull of } M).$$

Suppose that $x \in \cap\{ClU : U$ is n-hull of $M\}$ is not in $cl_{\theta^n} M$, i.e. $x \notin cl_{\theta^n} M$. By the Definition 2.2 there exists a chain of open neighborhoods of x, $U_1 \subset U_2 \subset \cdots \subset U_n$, such that $ClU_i \subset U_{i+1}$ for $i = 1, 2, \ldots, n-1$ and $ClU_n \cap M = \emptyset$. Now, a chain of open neighborhoods of $M, V_1 = X \setminus ClU_1 \supset V_2 = X \setminus ClU_2 \supset \cdots \supset V_n = X \setminus ClU_n$ is a n-hull $V = V_1$ of M such that $x \notin ClV_1$ since $ClV_1 \cap U_1 = \emptyset$. The proof of $cl_{\theta^n} M \supset \cap(ClU : U$ is n-hull of $M)$ is completed. □

Corollary 2.3. *A subset M of the space X is θ^n- closed if and only if*

$$M = \cap(ClU : U \text{ is n-hull of } M).$$

Definition 2.8. A mapping $f : X \to Y$ is said to be Θ^n-closed if $f(F)$ is Θ^n-closed for each Θ^n-closed subset $F \subset X$.

Lemma 2.4. *Let $f : X \to Y$ be a continuous mapping. The following conditions are equivalent:*

(a) *f is θ^n-closed,*
(b) *For every $B \subset Y$ and each Θ^n-open set $U \supseteq f^{-1}(B)$ there exists a Θ^n-open set $V \supseteq B$ such that $f^{-1}(V) \subset U$.*
(c) *f_{θ^n} is a closed mapping.*

Proof. The proof is similar to the proof of the corresponding theorem for closed mappings [4, p. 52]. □

Question. Let $f : X \to Y$ be a mapping. Under what conditions f is θ^n-closed?

Definition 2.9. Following [2, p. 48, (31)] we say that $f : X \to Y$ *is Θ^n-perfect* if and only if for every filter base $\mathcal{F}$ on X

$$f(\cap\{\mathrm{c1}_{\theta^n} F : F \in \mathcal{F}\}) \supseteq \cap\{\mathrm{c1}_{\theta^n} f(F) : F \in \mathcal{F}\}$$

Lemma 2.5. *If $f : X \to Y$ is Θ^n-perfect, then:*

(a) *For each* $A \subseteq X$, $\mathrm{c1}_{\theta^n} f(A) \subseteq f(\mathrm{c1}_{\theta^n} A\rangle$.
(b) *For each* θ^n*-closed* $\mathrm{A} \subseteq X$, $f(\mathrm{A})$ *is* θ^n*-closed.*

Definition 2.10. A function $f : X \to Y$ is *almost* Θ^n*-closed* if for any set $A \subseteq X$, $f(\mathrm{c1}_\theta A) = \mathrm{c1}_\theta f(\mathrm{A})$.

Lemma 2.6. *If* $f : X \to Y$ *is almost* Θ^n*-closed, then it is* Θ^n*-closed.*

Definition 2.11. A mapping $f : X \to Y$ is said to be skeletal (HJ) [7, p. 13] if for each open (regularly open) subset $U \subset X$ we have $\mathrm{Int} f^{-1}(ClU) \subset Clf^{-1}(U)$.

A mapping $f : X \to Y$ is said to have the *inverse property* if $Clf^{-1}(U) = f^{-1}(ClU)$ for every open set U in Y.

Each open mapping has the inverse property since a mapping $f : X \to Y$ is open if and only if $f^{-1}(ClB) = Clf^{-1}(B)$ or - equivalently - $\mathrm{Int} f^{-1}(B) = f^{-1}(\mathrm{Int} B)$ for every $B \subset Y$ [4, Exercise 1.4.C, p. 57].

Proposition 8. [7, p. 13]. *A mapping* $f : X \to Y$ *is HJ if and only if the counterimage of the boundary of each regularly open set is nowhere dense.*

A mapping $p : Y \xrightarrow{onto} X$ is said to be *irreducible* if for each closed subset A of Y $A \neq Y$ implies $Clp(A) \neq X$. A mapping $f : X \to Y$ is said to be *semi-open* provided $\mathrm{Int} f(U) \neq \emptyset$ for each non-empty open $U \subset X$. From Proposition 8 it follows the following result (see [7, 1.1, p. 27], [15, p. 236]).

Lemma 2.7. *Each semi-open, each open and each closed irreducible mapping is* HJ.

Proposition 9. *Let* $f : X \to Y$ *be a* Θ^n*-closed mapping and let* F *be a* Θ^n*-closed subset of* X*. The restriction* $g = f|F$ *is* Θ^n*-closed.*

Proof. By (c) of Lemma 2.4 it suffices to prove that $f_{\Theta^n}|i(F)$ is a closed mapping. Lemma follows since the restriction of closed e mapping f_{θ^n} onto a closed subset is closed. □

Lemma 2.8. *Let* $f : X \to Y$ *be a surjective mapping. If* F *is* Θ^n*-closed in* Y*, then* $f^{-1}(F)$ *is* Θ^n*-closed in* X.

Proof. Let us prove that $X \diagdown f^{-1}(F)$ is Θ^n-open. If x is a point of $X \diagdown f^{-1}$ (F), then $f(x) \in Y \diagdown F$. There exists an open hull U such that $f(x) \in U$ and $ClU \cap F = \emptyset$ since F is Θ-closed. Now $x \in f^{-1}(U)$ and $Clf^{-1}(U) \cap F = \emptyset$. We infer that $X \diagdown f^{-1}(F)$ is Θ^n-open. Hence, $f^{-1}(F)$ is Θ^n-closed by Definition 2.1. □

Definition 2.12. An S(n)-space $M, n > 0$, is *S(n)-closed* ([3]) if it is θ^n-closed in every S(n)-space in which it can be embedded.

Porter and Votaw [10] characterized $S(n)$-closed spaces by means of open $S(n)$-filters and $S(n)$-covers (for $n = 2$ it was done by Herrlich [6]). On the other hand Hamlett [5] proved that a Hausdorff space X is *H-closed* iff for every filter $\mathcal{F}$ on X $\mathrm{ad}_\theta\mathcal{F} \neq \emptyset$.

Proposition 10. [3, Proposition 2.1., p. 63] *Let $n \in \mathbb{N}^+$ and X be a space. Then the following conditions are equivalent:*

- (a) *For every open filter $\mathcal{F}$ on X $ad_{\theta^n}\mathcal{F} \neq \emptyset$;*
- (b) *For every filter $\mathcal{F}$ on X $ad_{\theta^n}\mathcal{F} \neq \emptyset$;*
- (c) *For every open $S(n)$-filter $\mathcal{F}$ on X $ad\mathcal{F} \neq \emptyset$;*
- (d) *For every $S(n-1)$-cover $\{U_\alpha\}$ of X there exist $\alpha_1, \alpha_2, \ldots, \alpha_k$ such that $X = \bigcup_{i=1}^{k} ClU_{\alpha_1}$.*

If X is an $S(n)$-space then the above conditions are equivalent to:
(e) *X is $S(n)$-closed.*

Proposition 11. *Let X be S(n)-closed space. Let $f : X \to Y$ be an HJ mapping and let F be a Θ^n-closed subset $F \subset X$. Then $f(F)$ is Θ^n-closed subset Y.*

Proof. **Step 1**. By Corollary 2.3 we have

$$F = \cap(ClU : U \text{ is } n\text{-hull of } F,$$

and let $\mathcal{V} = \{V_\lambda : \lambda \in \Lambda\}$ be a maximal family of n-hull containing F. Moreover, let $\mathcal{U} = \{U_\mu : \mu \in \Omega\}$ be a family of all n-hull of $f(F)$ such that there exists $V_\lambda \in \mathcal{V}$ such that $f(V_\lambda) \subset U_\mu$. For every n-hull $W \ni y$ we have $ClW \cap f(V_\lambda) \neq \emptyset$ since $ClW \cap f(V_\lambda) = \emptyset$ implies $Y \setminus ClW \supset f(V_\lambda)$, $Y \setminus ClW \in \mathcal{U}$ and $y \in Cl(Y \setminus ClW)$.

Step 2. Now, the set $W^* = \mathrm{Int}ClW$ is regularly open and, by virtue of Definition 2.11, we have

$$\mathrm{Int}f^{-1}(ClW^*) \subset Clf^{-1}(W^*).$$

From this and $f^{-1}(ClW^*) \cap V_\lambda \neq \emptyset$ it follows that $f^{-1}(W^*) \cap V_\lambda \neq \emptyset$ for each $V_\lambda \in \mathcal{V}$. The family $\mathcal{V}^* = \{V^*_\lambda : V^*_\lambda = f^{-1}(W^*) \cap V_\lambda\}$ has the finite intersection property. From the S(n)-closedness (Proposition 10) of X it follows that there exists adherent point $x \in \cap\{Cl_{\theta_n} V^*_\lambda : V^*_\lambda \in \mathcal{V}^*\}$. It is easy to prove that $x \in F$ and $f(x) \in \cap\{ClW : W$ is open set containing $y\}$. This means that $y = f(x)$ since Y is a Hausdorff space. Hence, $f(F) \supset cl_{\theta^n} f(F) = \cap(ClV : V$ is n-hull of $f(F)$. The proof of Proposition is completed. □

Corollary 2.9. *Each semi-open (open, closed irreducible) and each mapping with the inverse property is Θ^n-closed .*

Lemma 2.10. *If X is S(n)-closed, then every family $\{A_\mu,\ \mu \in \Omega\ \}$ of θ^n-closed subsets of X with the finite intersection property has a non-empty intersection $\cap\{A_\mu,\ \mu \in \Omega\ \}$.*

Proof. Let X be S(n)-closed and let $\{A_\mu,\ \mu \in \Omega\ \}$ be a family of θ^n-closed subsets of X with the finite intersection property. By (b) of Proposition 10 we infer that $\mathrm{ad}_{\theta^n}\{A_\mu, \mu \in \Omega\} \neq \emptyset$, i.e. $\cap\{Cl_{\theta^n} A_\mu, \mu \in \Omega\} \neq \emptyset$ 2.3. But $Cl_{\theta^n} A_\mu = A_\mu$ since $\{A_\mu,\ \mu \in \Omega\ \}$ is a family of θ^n-closed subsets of X. Finally we infer that $\cap\{A_\mu, \mu \in \Omega\} \neq \emptyset$. □

Lemma 2.11. *If $(X,\ \tau)$ is $S(n)$-closed space then the space $(X,\ \tau_{\theta^n})$ is quasi-compact.*

Proof. Let $(X,\ \tau)$ be an $S(n)$-closed space and let us prove that $(X,\ \tau_{\theta^n})$ is quasi-compact. For every filter $\mathcal{F} = \{F : F \in \mathcal{F}\}$ of closed sets on $(X,\ \tau_{\theta^n})$ we have the family $\{Cl_{\Theta_n} i^{-1}(F) : F \in \mathcal{F}\}$ with non-empty intersection $\cap\{Cl_{\Theta_n} i^{-1}(F) : F \in \mathcal{F}\}$ since $(X,\ \tau)$ is $S(n)$-closed space. It is clear that $\cap\{F : F \in \mathcal{F}\} \neq \emptyset$ since $i(Cl_{\Theta_n} i^{-1}(F)) = ClF = F$. Hence, $(X,\ \tau_{\theta^n})$ is quasi-compact. □

Following Porter and Thomas [11] (for $n = 1$ they introduced quasi-H-closed spaces) the spaces satisfying the equivalent conditions $(a) - (d)$ will be called *quasi* $S(n)$*-closed*.

An open set U of a topological space $(X,\ \tau)$ is *regularly open* if $U = \mathrm{Int}Cl\ U$. The topology on X which has as a basis the set of regularly open sets of $(X,\ \tau)$ is denoted by τ_s; it is the *semiregularization* of τ and $(X,\ \tau)$ is *semiregular* if $\tau_s = \tau$.

The set $F \subset X$ regularly closed if $F = Cl\mathrm{Int}F$. A space X is *almost regular* if for each regularly closed set $F \subset X$ and each point $x \in X \diagdown F$ there are disjoint open sets $U \ni x$ and $V \supset F$. Moreover, the space $(X,\ \tau)$ is almost regular if $(X,\ \tau_s)$ is regular.

For a *space* X and $n \in \mathbb{N}^+$ denote by $o_n(X)$ the (ordinal) number of iterations of the θ^n-closure to get a Kuratowski operator (it will be the closure in $\tau_{\theta'^{\prime}}$). We call $o_n(X)$ 274^n*-order* of X. By Theorem 1.2 of [3] $o_1(X) = 1$ iff X is almost regular. We shall sometimes need this result that in an almost regular space X, $Cl_\Theta A$ is $\Theta-$closed (i.e. $Cl_\Theta(Cl_\Theta A) = Cl_\Theta A$ [8].

Let X be a set and $\mathcal{P}(X)$ its power set. A *Kuratowski Closure Operator* is an assignment cl: $\mathcal{P}(X) \to \mathcal{P}(X)$ with the following properties:

1. $cl(\emptyset) = \emptyset$ (Preservation of Nullary Union);
2. $A \subset cl(A)$ (Extensivity);
3. $cl(A \cup B) = cl(A) \cup cl(B)$ (Preservation of Binary Union);
4. $cl(cl(A)) = cl(A)$ (Idempotence).

If the last axiom, Idempotence, is omitted, then the axioms define a *Preclosure Operator*. It is clear that θ^n*-closure* $Cl_{\theta^n}M$ is a preclosure operator.

A consequence of the third axiom is: $A \subseteq B \Longrightarrow cl(A) \subseteq cl(B)$ (Preservation of Inclusion).

Clearly, (4) implies the following result.

Theorem 2.12. *Let* $(X,\ \tau)$ *be S(n)-space. If* θ^n*-closure* $Cl_{\theta^n}M$ *is Kuratowski Closure Operator i.e.* $Cl_{\theta^n}(Cl_{\theta^n}(A)) = Cl_{\theta^n}(A)$*, then* $(X,\ \tau)$ *is* $S(n)$*-closed space if and only if the space* $(X,\ \tau_{\theta^n})$ *is quasi-compact.*

Proof. The "if" part. Let the space $(X,\ \tau_{\theta^n})$ be quasi-compact. For every filter $\mathcal{F}$ on X we have the family $\{Cl_{\Theta_n}F : F \in \mathcal{F}\}$. Using the mapping $i : (X,\ \tau) \to (X,\ \tau_{\theta^n})$ from Proposition 6 we obtain the family $\{i(Cl_{\Theta_n}F) : F \in \mathcal{F}\}$ which is the filter of closed sets in $(X,\ \tau_{\theta^n})$. From the quasi-compactness of $(X,\ \tau_{\theta^n})$ it follows that $\cap\{i(Cl_{\Theta_n}F) : F \in \mathcal{F}\} \neq \emptyset$. Clearly, $\cap\{Cl_{\Theta_n}F : F \in \mathcal{F}\} = \mathrm{ad}_{\theta^n}\mathcal{F} \neq \emptyset$ (see Definition 2.3). By (e) of Proposition 10 we infer that $(X,\ \tau)$ is $S(n)$-closed space.

The "only if" part. See 2.11. □

Theorem 2.13. $S(n)$*-closed space* $(X,\ \tau)$ *is quasi-compact if every closed subset of* $(X,\ \tau)$ *is* θ^n*- closed.*

Proof. The identity mapping $i : (X,\ \tau) \to (X,\ \tau_{\theta^n})$ is continuous by Proposition 6. If every closed subset of $(X,\ \tau)$ is θ^n- closed, then the identity i is closed. Thus, $i : (X,\ \tau) \to (X,\ \tau_{\theta^n})$ is the homeomorphism. This means that $(X,\ \tau)$ is quasi-compact since $(X,\ \tau_{\theta^n})$ is quasi-compact by Theorem 2.12. □

3. Inverse systems of S(n)-closed spaces - inverse system $\mathbf{X}_{\Theta^n}$

This section is the main section of this paper. We shall use the notion of inverse systems as in the book [4, pages 135-144] and we start with the following elementary proposition.

Proposition 12. *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an inverse system of S(n)-spaces. If the projections* $p_a : \lim \mathbf{X} \to \mathbf{X}_a, a \in A$*, are surjections, then* $\lim \mathbf{X}$ *is S(n)-space.*

Proof. Let x, y be a pair of different points in $\lim \mathbf{X}$. There exists an $a \in A$ such that $p_b(x) \neq p_b(y)$ for every $b \geq a$. By virtue of (a) Definition 2.5 it suffices to prove that every pair of distinct points of X are S(n)-separated (Definition 2.4). Let us prove that x is S(n)-separated from y, i,e.,that $x \notin Cl\ _{\theta^n}\{y\}$. From the fact that X_a is S(n)-space it follows that $p_a(x)$ is S(n)-separated from $p_a(y)$. This means that there exists a n-hull U 2.2 of $p_a(x)$, i.e., there exists a family of open sets $U_1, U_2, \ldots, U_n = U$ such that

$p_a(x) \in U_1$, $ClU_i \subset U_{i+1}$ for $i = 1, \ldots, n-1$ and $p_a(y) \notin ClU$. It is clear that $p_a^{-1}(U_1), p_a^{-1}(U_2), \ldots, p_a^{-1}(U_n) = p_a^{-1}(U)$ is the n-hull of x and $y \notin Clp_a^{-1}(U)$. Hence $\lim \mathbf{X}$ is a S(n)-space. □

Theorem 3.1. *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an inverse system with HJ mappings p_{ab}. If the projections $p_a : \lim \mathbf{X} \to X_a, a \in A$, are surjections, then they are HJ mapping and, consequently, Θ^n-closed .*

Proof. By Proposition 11 a mapping $f : X \to Y$ is HJ if and only if the counterimage of the boundary of each regularly open set is nowhere dense. Suppose that p_a is not HJ. Then there exist a regularly open set U_a in X_a such that the boundary of $p_a^{-1}(U_a)$ contains an open set U. From the definition of a base in $\lim \mathbf{X}$ it follows that there is a $b \geq a$ and an open set U_b in X_b such that $p_b^{-1}(U_b) \subset U$. It is clear that $U_b \subset Bd\ p_{ab}^{-1}(U_a)$. This is impossible since p_{ab} is HJ. Hence, the projections $p_a, a \in A$, are HJ. From Proposition 11 it follows that p_a is Θ^n-closed. □

For every inverse system $\mathbf{X} = \{X_a, p_{ab}, A\}$ of S(n)-closed spaces we shall introduce the inverse system $\mathbf{X}_{\Theta^n}$. Namely, for every space X_a there exists the space $(X_a)_{\Theta^n}$ which is defined in Definition 2.6 as the space (X, τ_{θ^n}). Moreover, for every mapping $p_{ab} : X_b \to X_a$ there exists the mapping $(p_{ab})_{\Theta^n}$ (see Definition 2.7 and Lemma 2.2). The transitivity condition

$$(p_{ab})_{\Theta^n}(p_{bc})_{\Theta^n} = (p_{ac})_{\Theta^n}$$

follows from the commutativity of the diagram 2.1. This means that we have the following result.

Proposition 13. *For every inverse system $\mathbf{X} = \{X_a, p_{ab}, A\}$ of S(n)-closed spaces there exists the inverse system $\mathbf{X}_{\Theta^n} = \{(X_a)_{\Theta^n}, (p_{ab})_{\Theta^n}, A\}$ such that the following diagram commutes*

$$\begin{array}{ccccccc}
X_a & \overset{p_{ab}}{\longleftarrow} & X_b & \overset{p_{bc}}{\longleftarrow} & X_c & \ldots & \lim \mathbf{X} \\
\downarrow i_a & & \downarrow i_b & & \downarrow i_c & & \downarrow i \\
(X_a)_{\Theta^n} & \overset{(p_{ab})_{\Theta^n}}{\longleftarrow} & (X_b)_{\Theta^n} & \overset{(p_{bc})_{\Theta^n}}{\longleftarrow} & (X_c)_{\Theta^n} & \ldots & \lim \mathbf{X}_{\Theta^n}
\end{array}$$

where i and each i_a is the identity for every $a \in A$.

Let us recall that the continuity of the mapping i was proved in Proposition 6.

Proposition 14. *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an inverse system. There exists a mapping $p_\Theta : (\lim \mathbf{X})_{\Theta^n} \to \lim \mathbf{X}_{\Theta^n}$ such that $i = p_\Theta i_\Theta$, where $i_\Theta : \lim \mathbf{X} \to (\lim \mathbf{X})_{\Theta^n}$ is the identity.*

Proof. By Definition 2.1 for each $a \in A$ there is $(p_a)_\Theta : (\lim \mathbf{X})_\Theta \to (X_a)_\Theta$. This mapping is continuous (Lemma 2.2). The collection $\{(p_a)_\Theta : a \in A\}$

induces a continuous mapping $p_\Theta : (\lim \mathbf{X})_\Theta \to \lim \mathbf{X}_\Theta$. Hence we have the following diagram.

$$\begin{array}{ccc} \lim \mathbf{X} & \overset{id}{\to} & \lim \mathbf{X} \\ \downarrow i & & \downarrow i_\Theta \\ \lim \mathbf{X}_{\Theta^n} & \overset{p_\Theta}{\leftarrow} & (\lim \mathbf{X})_{\Theta^n} \end{array}$$

□

In the sequel we shall use the following results.

Theorem 3.2. [13, Theorem 3, p. 206]. *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an inverse system of quasi-compact non-empty* T_0 *spaces and closed bonding mapping* p_{ab}. *Then* $\lim \mathbf{X}$ *is non-empty.*

Theorem 3.3. [13, Theorem 5, p. 208].*Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an inverse system of quasi-compact* T_0 *spaces and closed bonding mapping* p_{ab}. *Then* $\lim \mathbf{X}$ *is quasi-compact.*

Now, we shall prove the following result.

Lemma 3.4. *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an inverse system of quasi-compact non-empty* T_0 *spaces and closed surjective bonding mapping* p_{ab}. *Then the projections* $p_a : \lim \mathbf{X} \to \mathbf{X}_a, a \in A$, *are surjective and closed.*

Proof. Let us prove that the projections p_a are surjective. For each $x_a \in X_a$ the sets $Y_b = p_{ab}^{-1}(x_a)$ are non-empty closed sets. This means that the system $\mathbf{Y} = \{Y_b, p_{bc}|Y_c, a \leq b \leq c\}$ satisfies Theorem 3.2 and has a non-empty limit. For every $y \in Y$ we have $p_a(y) = x_a$. Hence, p_a is surjective. It remains to prove that p_a is closed. It suffices to prove that for every $x_a \in X_a$ and every neighborhood U of $p_a^{-1}(x_a)$ in $\lim \mathbf{X}$ there exists an open set U_a containing x_a such that $p_a^{-1}(U_a) \subset U$. For every $x \in p_a^{-1}(x_a)$ there is a basic open set $p_{a(x)}^{-1}(U_{a(x)})$ such that $x \in p_{a(x)}^{-1}(U_{a(x)}) \subset U$. From the quasi-compactness of $p_a^{-1}(x_a)$ it follows that there exists a finite set $\{x_1, \dots, x_n\}$ of the points of $p_a^{-1}(x_a)$ such that $\{p_{a(x_1)}^{-1}(U_{a(x_1)}), \dots, p_{a(x_n)}^{-1}(U_{a(x_n)})\}$ is an open cover of $p_a^{-1}(x_a)$. Let $b \geq a(x), a(x_1), \dots, a(x_n)$ and let $U_b = \cup\{p_{a(x_1)b}^{-1}(U_{a(x_1)}), \dots, p_{a(x_n)b}^{-1}(U_{a(x_n)})\}$. It follows that $p_b^{-1}(U_b) \subset U$ and $p_{ab}^{-1}(x_a) \subset U_b$. From being closed p_{ab} it follows that there is an open set U_a containing x_a such that $p_{ab}^{-1}(U_a) \subset U_b$. Finally, $p_a^{-1}(U_a) \subset U$. □

The following Theorem is the main result of this section.

Theorem 3.5. *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an inverse system of non-empty S(n)-closed spaces and* Θ^n*-closed bonding mapping* p_{ab}. *Then* $\lim \mathbf{X}$ *is non-empty. Moreover, if* p_{ab} *are surjections, then the projections* $p_a : \lim \mathbf{X} \to X_a$, $a \in A$, *are surjections.*

Proof. Consider the following diagram

$$\begin{array}{ccccccc} X_a & \overset{p_{ab}}{\leftarrow} & X_b & \overset{p_{bc}}{\leftarrow} & X_c & \ldots. & \lim \mathbf{X} \\ \downarrow i_a & & \downarrow i_b & & \downarrow i_c & & \downarrow i \\ (X_a)_{\Theta^n} & \overset{(p_{ab})_{\Theta^n}}{\longleftarrow} & (X_b)_{\Theta^n} & \overset{(p_{bc})_{\Theta^n}}{\longleftarrow} & (X_c)_{\Theta^n} & \ldots. & \lim \mathbf{X}_{\Theta^n} \end{array}$$

from Proposition 13. By Theorem 2.12 each $(X_a)_{\Theta^n}$ is a quasi-compact T_1 space. Furthermore, each mapping $(p_{ab})_{\Theta^n}$ is closed by c) of Lemma 2.4 since p_{ab} is Θ^n-closed (see Definition 2.8). This means that the inverse system $\mathbf{X}_\Theta = \{(X_a)_\Theta,\ (p_{ab})_\Theta, A\}$ satisfies the conditions of Theorem 3.2. It follows that $\lim \mathbf{X}_\Theta$ is non-empty. This implies that $\lim \mathbf{X}$ is non-empty. Further, if $p_{ab}, b \geq a$, are onto mappings, then for each $x_a \in X_a$ the sets $Y_b = p_{ab}^{-1}(x_a)$ are non-empty Θ^n-closed sets (Lemma 2.8). This means that the system $\mathbf{Y}_\Theta = \{(Y_b)_\Theta, (p_{bc})_\Theta|(Y_c)_\Theta, a \leq b \leq c\}$ satisfies Theorem 3.2 and has a non-empty limit. This means $\mathbf{Y} = \{Y_b, p_{bc}|Y_c, a \leq b \leq c\}$ has a non-empty limit. For every $y \in Y$ we have $p_a(y) = x_a$. □

Lemma 3.6. *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an inverse system of S(n)-closed spaces and* Θ^n*-closed surjective bonding mapping* p_{ab}*. Then the projections* $p_a : \lim \mathbf{X} \to \mathbf{X}_a, a \in A$*, are* Θ^n*-closed if and only if the mapping* p_Θ : $(\lim \mathbf{X})_{\Theta^n} \to \lim \mathbf{X}_{\Theta^n}$ *from Proposition* 14 *is a homeomorphism.*

Proof. The "if" part. Now we have the following diagram

$$\begin{array}{ccccccc} X_a & \overset{p_{ab}}{\leftarrow} & X_b & \overset{p_{bc}}{\leftarrow} & X_c & \ldots. & \lim \mathbf{X} \\ \downarrow i_a & & \downarrow i_b & & \downarrow i_c & & \downarrow i \\ (X_a)_{\Theta^n} & \overset{(p_{ab})_{\Theta^n}}{\longleftarrow} & (X_b)_{\Theta^n} & \overset{(p_{bc})_{\Theta^n}}{\longleftarrow} & (X_c)_{\Theta^n} & \ldots. & \lim \mathbf{X}_{\Theta^n} = (\lim \mathbf{X})_{\Theta^n} \end{array}$$

where i and each i_a is the identity for every $a \in A$. Each projection q_a : $(\lim \mathbf{X})_{\Theta^n} \to (X_a)_{\Theta^n}$ is closed (Lemma 3.4). From (a) and (c) of Lemma 2.4 it follows that the projection $p_a : \lim \mathbf{X} \to \mathbf{X}_a$ is Θ^n-closed for every $a \in A$.

The only "if" part. Suppose that the projections $p_a : \lim \mathbf{X} \to \mathbf{X}_a, a \in A$, are Θ^n-closed. Let us prove that p_Θ is a homeomorphism. It suffice to prove that p_Θ is closed. Let $F \subset (\lim \mathbf{X})_{\Theta^n}$ be closed. This means that F is Θ^n-closed in $\lim \mathbf{X}$. For each $a \in A$ the set $p_a(F)$ is Θ^n-closed since the projections p_a are Θ^n-closed. Now, $i_a p_a(F)$ is closed in $(X_a)_{\Theta^n}$. We have the collection $\{q_a^{-1} i_a p_a(F) : a \in A\}$ having the finite intersection property. It is clear that $p_\Theta(F) = \cap\{q_a^{-1} i_a p_a(F) : a \in A\}$ and that $\cap\{q_a^{-1} i_a p_a(F) : a \in A\}$ is closed in $\lim \mathbf{X}_{\Theta^n}$. Hence, p_Θ is closed and, consequently, a homeomorphism. □

Question. Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an inverse system of S(n)-closed spaces Under what conditions $\lim \mathbf{X}$ is S(n)-closed?

Now we shall prove some partial answers to this question.

Theorem 3.7. *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an inverse system of non-empty S(n)-closed spaces such that* θ^n*-closure* $Cl_{\theta^n}M$ *is Kuratowski Closure Operator i.e.* $Cl_{\theta^n}(Cl_{\theta^n}(A)) = Cl_{\theta^n}(A)$. *If the projections* $p_a : \lim \mathbf{X} \to \mathbf{X}_a, a \in A$, *are* Θ^n*-closed, then* $X = \lim \mathbf{X}$ *is non-empty and S(n)-closed.*

Proof. Using Theorem 3.5 we infer that $\lim \mathbf{X}$ is non-empty. By the only if part of Lemma 3.6 we infer that the mapping $p_\Theta : (\lim \mathbf{X})_{\Theta^n} \to \lim \mathbf{X}_{\Theta^n}$ from Proposition 14 is a homeomorphism, i.e. that $\lim \mathbf{X}_{\Theta^n} = (\lim \mathbf{X})_{\Theta^n}$. From Theorem 3.3 it follows that $\lim \mathbf{X}_{\Theta^n}$ is quasi-compact. Moreover, from $\lim \mathbf{X}_{\Theta^n} = (\lim \mathbf{X})_{\Theta^n}$ we infer that $(\lim \mathbf{X})_{\Theta^n}$ is quasi-compact. Finally, (a) of Theorem 2.12 completes the proof. □

Theorem 3.8. *If* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *is an inverse system of non-empty S(n)-closed spaces* X_a *and HJ mappings* p_{ab} *such that* θ^n*-closure* $Cl_{\theta^n}M$ *is Kuratowski Closure Operator i.e.* $Cl_{\theta^n}(Cl_{\theta^n}(A)) = Cl_{\theta^n}(A)$*, then* $X = \lim \mathbf{X}$ *is non-empty and S(n)-closed.*

Proof. Using Theorem 3.5 we infer that $\lim \mathbf{X}$ is non-empty. Then from Theorem 3.1 it follows that the projections $p_a : \lim \mathbf{X} \to \mathbf{X}_a, a \in A$, are HJ mapping and, consequently, Θ^n-closed. Theorem 3.7 completes the proof. □

Corollary 3.9. *If* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *is an inverse system of non-empty S(n)-closed spaces* X_a *and semi-open* (*open, closed irreducuble*) *mappings* p_{ab} *such that* θ^n*-closure* $Cl_{\theta^n}M$ *is Kuratowski Closure Operator i.e.* Cl_{θ^n} $(Cl_{\theta^n}(A)) = Cl_{\theta^n}(A)$ *, then* $X = \lim \mathbf{X}$ *is non-empty and S(n)-closed.*

We say that an inverse system $\mathbf{Y} = \{Y_a, q_{ab}, A\}$ is a subsystem of $\mathbf{X} = \{X_a, p_{ab}, A\}$ if $Y_a \subset X_a, a \in A$, and $q_{ab} = p_{ab}|Y_b$. In this case we shall write $\mathbf{Y} = \{Y_a, p_{ab}|Y_b, A\}$.

Proposition 15. *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an inverse system of non-empty S(n)-closed spaces and* Θ^n*-closed bonding mapping* p_{ab}. *If* $\mathbf{Y} = \{Y_a, p_{ab}| Y_b, A\}$ *is an inverse subsystem of* Θ^n*-closed subsets* $Y_a \subset X_a, a \in A$*, then* $\lim \mathbf{Y}$ *is non-empty.*

Proof. Now $\{(p_a)^{-1}_{\Theta^n} i_a(Y_a) : a \in A\}$ is a family of closed sets in $\lim \mathbf{X}_{\Theta^n}$ with the finite intersection property. From the quasi-compactness of $\lim \mathbf{X}_{\Theta^n}$ (Theorem 3.3) it follows that $\cap\{(p_a)^{-1}_{\Theta^n} i_a(Y_a) : a \in A\}$ is non-empty. Hence $\lim \mathbf{Y} \neq \emptyset$ since $\lim \mathbf{Y} = \mathbf{i}^{-1}(\cap\{(p_a)^{-1}_{\Theta^n} i_a(Y_a) : a \in A\})$. □

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