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HAPPY BIRTHDAY MIRJANA !

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ABSTRACT. This issue of Sarajevo Journal of Mathematics is devoted to our editor in chief Acad. Prof. Dr. Mirjana Vuković on the occasion of her jubilee. Here we will give a brief overview of her life and work.

1. MIRJANA'S CHILDHOOD AND EDUCATION

As a long-time friend I learned many details from Mirjana's life.

Mirjana Vuković was born in 1948 in Fojnica in BiH, although her family lived in Sarajevo. Very quickly, the young family headed north, first to Murska Sobota (Slovenia), then to Varaždin (Croatia), and thereafter to Maribor (Slovenia). Although Mirjana was born in a difficult time, after the Second World War, it can be said that she was born in a family that gave her a lot of love. She has often and mentioned that the single biggest influence on her successful career came from her mother Mila.

As a little girl, Mirjana felt she was always a bit different from her peers. Before the age of 6, she didn't play with toys and with other children, she liked to listen to her mother read to her and she enjoyed drawing.

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She spent the first twelve years of her life in Varaždin and Maribor, where she completed 6 years of elementary school. After that they returned to Sarajevo where Mirjana completed elementary school and graduated from the *Braća Ribar High School* (today the *Third Gymnasium*) and according to her teachers, she was one of the best high school students.

Both parents were encouraging her early intellectual development and later supported her desire to study mathematics though she excelled in all subjects and had a talent even for writing, painting, etc.

She enrolled in mathematics studies at the Department of Mathematics of the Faculty of Science, University of Sarajevo. She graduated summa cum laude in her class, winning all the silver and gold medals of the University and a prize scholarship from the *Hasan Brkić University Fund*, which she received as the nominee of her professors, academician *Mahmut Bajraktarević* and *Šefkija Raljević*, then Dean of the Faculty. Already after the first year of study they developed Mirjana's mathematical talents, commenting she would become an assistant if she continued at her pace. Not only did she become an assistant to the acad. M. Bajraktarević, (and later Šefkija Raljević), but she also succeeded him as the professor of Complex Analysis.

Immediately after completing her studies, M. Vuković started her academic career at the Mathematics Department of the Faculty of Science in Sarajevo and spent her entire working life there, including the war years. She also acquired knowledge at other, most prestigious universities in the world, such as *Moscow State University – Lomonosov* (Moscow, 1975/76), and then *Pierre et Marie Curie* University in Paris (December 1976), where she went for years after her first study stay and collaborated with the famous prof. Marc Krasner[1]. That is why, when she talks about her education, Mirjana often says: my three universities, referring to *Sarajevo University*, Moscow's *Lomonosov* and Paris' *Pierre et Marie Curie*. At the Faculty of Science in Sarajevo she earned the titles of master's degree (1975) and doctorate (1979) in mathematical sciences and rose through the academic ranks becoming a full professor in 1989.

2. About Mirjana's Research Work

Mirjana's scientific interests are concentrated around several important and contemporary areas of mathematical analysis and modern algebra. Her scientific papers, which are of a fundamental-theoretical character, can be divided into two circles. The first circle includes papers belonging to the field of mathematical analysis. While preparing her doctoral thesis, under the mentorship of academician Manojlo Maravić, within this group of works she studied the summability of

[1] one of the leading mathematicians in the field of algebra and number theory who was one of the most important mathematicians of his time, the holder of the title of *Officier des Palmes Académique* and the first laureate of the *Paul Doisteau – Emil Blutet* award (1958) of the French Academy of Sciences.

multiple Fourier series and expansions in eigenfunctions of the Laplace operator, Karamata's theory, proved several Tauber theorems and one convexity theorem for the G_h^{κ} - method of summability that was, at that time, one of the three convexity theorems – next to Riesz's and Maravić's, hers was the third.

A special note however belongs to the second group of her works, those in the field of abstract algebra, which represent the crown of her scientific work in which, first, together with the famous French mathematician M. Krasner, she gave an answer to a long open question: under what conditions are graded structures (groups, rings, modules) closed with respect to direct sum and direct product? Through that answer, they arrived at graded structures, more general than those of Bourbaki-Krasner, which they called paragraded structures (groups, rings, modules). They first appeared in their joint works *Structures paragraduées* I, II, III [1-3], published in the prestigious publication *Proceedings of The Japan Academy* of the Japanese Academy of Sciences, based on the reviews of one of the greatest and most influential Japanese mathematicians of the time, academician *Shokichi Iyanaga*, and then in the scientific monograph *Structures paragraduées* (groups, rings, modules), which was published in the well-known monographic edition *Queen's Papers in Pure and Applied Mathematics* from Canada [4], which gained significant publicity with its announcement in the book *II mondo Krasneriano* [5], by the famous Canadian mathematician *Paulo Ribenboim*.

While the first group of works is important for mathematical analysis, and can be interesting for solving problems of mathematical physics, the second group of works (those in algebra) is completely abstract and has not yet found application. But these among her discoveries can serve as an inspiration for scientists all over the world for the further development of graded structures.

Now we will say something about the works from that group.

Discovering paragraded structures, Krasner - Vuković's assumption that it is possible to find graded structures, closed with respect to the direct product and the direct sum, has been confirmed. Successful study of Bourbaki - Krasner's graded structures problems opened the way to paragraded structures and brought attention to Mirjana. In one of her papers in the field of paragraded structures Mirjana discussed the primary decomposition in the case of general graded modules - moduloids, a generalization of already published results for general graded rings - anneids. After proving the existence and uniqueness of primary decomposition of moduloids, she briefly turned our attention to Krull's theorem and to the existence of the primary decomposition of Krasner-Vuković paragraded rings [6].

In several of her papers she discussed the prime and Jacobson radicals [7], [9], [12], and the general theory of radicals [8].

In her next joint paper with her pupil Ilić-Georgijević [9], they discussed the general theory of radicals of paragraded rings (also introduced for the first time in these paper) and characterized paragraded normal radicals. It is well known that

the ADS-theorem overcomes the problem of the relation "*being an ideal*" not being transitive for associative rings. They proved a version of the ADS-theorem for associative paragraded rings, i.e., that for any paragraded radical α (in the sense of *Kurosh-Amitsur*) and any associative paragraded ring R, if I is a homogeneous ideal of R, then $\alpha(I)$ is a homogeneous ideal of R. They also studied special paragraded radicals of paragraded rings. In addition, starting from the well-known fact that any special radical of graded ring can be described by an appropriate class of modules over that ring, they have shown that all special paragraded radicals of paragraded rings can be described by an appropriate class of their paragraded modules in an analogous manner as in the case of graded rings.

Mirjana, as many algebraists, was attracted to the *Wedderburn-Artin* theorem and she extends it to the paragraded rings in her joint paper with E. Ilić-Georgijević [10]. Following the methods known from the abstract case, they first proved the density theorem and observed the matrix rings whose entries are from paragraded rings. However in order to arrive at the desired structures theorem, they introduced the notion of Jacobson radical of a paragraded ring and proved some properties which are analogous to the abstract case. In the process, they studied the faithful and irreducible modules over noncommutative paragraded rings and proved the paragraded version of the well-known Schur Lemma.

In order to tear M. Krasner from oblivion and point out his importance, Mirjana started her paper [11] with a short historical development of graduation which begins with Krasner's famous notion of a corpoid, introduced in 1940s and general graded groups in Krasner's sense, more general than Bourbaki's. For example, Krasner's graded structures are more general than graded structures of Bourbaki since they do not require the associativity nor the commutativity nor the unitarity in the set of grades. In addition, it presented some results in the theory of Krasner-Vuković's para- and extra- graded groups including examples of paragraduations which are and which are not graduations, and some proofs of statements that were not given earlier and finally provided the missing step in the proof of the result.

In the paper [12] Mirjana studied paragraded modules over noncommutative paragraded rings and, as the main result, proved the paragraded version Schur's lemma.

In the next paper [13] Mirjana presented prime and Jacobson radicals, discussed the general Kurosh-Amitsur theory of radicals of paragraded rings, established that the ADS (Anderson, Divinsky, and Suli'nski) theorem holds for paragraded rings, characterized paragraded normal radicals, and proved that all special paragraded radicals of paragraded rings can be described by appropriate classes of their graded modules. The results of this paper were presented at the International Algebraic Conference dedicated to the 110th anniversary of birth A.G. Kurosh, Moscow, 2018.

The paper [14] begins with a note about Aleksander V. Mikhalev and a short introduction of some historic facts about "graded structures that are old as well

new" by M. Krasner. Later, Mirjana gives a panoramic view of more general Krasner graded groups and introduces Krasner-Vuković's paragraded groups and then concludes with some results in the theory of paragraded groups.

The purpose of the paper [15] was to introduce two versions of paragraded Brown-McCoy radicals: the Brown-McCoy radical and the large Brown McCoy radical of paragraded rings, and then, inspired by Halberstadt results about Jacobson's radicals of graded rings, to prove that the large Brown McCoy radical of paragraded ring coincides with the largest homogeneous ideal contained in the classical Brown McCoy radical of the ring.

In addition to papers in analysis and algebra, we should definitely mention her works on mathematicians, her professors: *Vera Šnajder, Mahmut Bajraktarević, Šefkija Raljević, Manojlo Maravić, Branislav Martić* – students of the Belgrade school of mathematics [18], as well as the latest book *Mathematicians – academicians* [19], which the reviewer, academician *Dejan Milošević*, describes as follows: "*The main, third chapter, forms the backbone of the book. By skilfully combining biographical data, personal memories and a knowledgeable description of academicians' scientific contribution, the author, academician Mirjana Vuković, has created a work of special significance, one which will be read with pleasure even by those for whom mathematics is not the main preoccupation in life. And it was precisely the extremely broad knowledge of mathematics, which the author of this book chose as her life's path, that enabled academician Mirjana Vuković to write expertly about the various areas of mathematics that these academicians dealt with*."

These works of Mirjana Vuković, which, in addition to biographical data, present the scientific activities of the mathematicians academicians, belong to the history of mathematics [18], [19].

It is safe to say that Mirjana Vuković, in addition to her connection with the famous Kurosh's Russian school of algebra, is also linked by double threads to one of the oldest and most famous schools of mathematics in the world – the French School of Mathematics: through the Belgrade School of Analysis of Mihailo Petrović Alas and Manojlo Maravić – her PhD mentor – as well as through algebra and the renowned Marc Krasner.

3. About Mirjana and Her Life Dedicated to Mathematics

One would not have a complete picture of Mirjana if we did not consider her human aspect. During the long term acquaintance discussion, and even a joint work with her I have gotten to know her as a person who is solid, very correct, an example of an intelectual person – but also a person who could listen and truly want to help, even raise voice against injustice. As we have already noted she knew early on that she should do what she loved – and what she loved was mathematics. So, she fell in love with mathematics very early and although it was her greatest love, Mirjana was a refined intellectual with a wide interest in art, especially literature

and painting, but also physics – she completed almost 3 years of studying physics, as well as the other natural sciences.

Mirjana is the author and co-author of over 120 scientific and professional papers and more than 10 books and textbooks. We will highlight some of them: the already mentioned monograph *Structures paragraduées* (*groupes, anneaux, modules*) [4], which can be found in more than 154 libraries worldwide, and is also included in the *Open Library* among the 105 most important books on commutative algebra, *World Cat, Amazon Company*; then *Theory of groups and representations with applications in physics* [16], "*in which some areas appear for the first time in literature in South Slavic languages*" (from the review by Academician V. Perić); *Algebra – Theory of Groups* (*Review of theory and problems*) written in co-authorship with Acad. V. Perić [17]; as well as her latest book *Mathematicians – academicians* [19], and the scientific journal *Sarajevo Journal of Mathematics*, of which, for a while, she was the co-editor-in-chief (2014-2019), and since 2019 she is its editor-in-chief.

She was a researcher, visiting researcher, and visiting professor at several prestigious universities and institutes such as *Lomonosov – Moscow State University* in Moscow and *Pierre et Marie Curie* University in Paris, *Institut Joseph Fourier* in Grenoble, *Technische Universität* – Wien in Vienna, as well as *Fields Institute*, Toronto, Canada; *Charles University*, Prague, Czech Republic; Tsukuba University, Tsukuba, Japan; *Mathematics Institute of SASA*, Belgrade, Serbia; *Johannes Kepler Universität*, Linz, Austria, etc.

She participated as the head researcher or team leader in about 20 scientific-research projects, including 4 foreign ones, two of which were *Central European Research Support Schemes* and on two she was a co-leader, namely: *Paragraded Structures and their Applications to Non-Archimedian Analysis*, at the *Institute Joseph Fourier*, Grenoble (with Prof. A. Panchishkin) and on the project *Connections between Krasner – Vuković paragraded structures* (*groups, rings, modules*) and *Lie superalgebras* in Maribor (with Prof. D. Pagon) which were financed first by the *Rhône Alpes region TEMPRA – PECO* (2001), and second from the funds of *JoinEU-SEE – ERASMUS Mundus Project Partnership* (2013). According to the co-leader D. Pagon: *the goal was to connect the results of two magnificent schools of algebra – French on the one hand and Russian – Moscow on the other*.

Mirjana Vuković made a significant contribution in all spheres of the development of mathematics: as a scientist with notable scientific results, as a lecturer and teacher of numerous generations, and as an author of exceptionally well-written textbooks, books, monographs, etc. (F. Vajzović).

She was elected for the numerous academic functions at the University of Sarajevo, the Department of Mathematics, and the Faculty of Science, of which we will mention only the three most important, to which she was elected before the war: vice dean for science and education at the Faculty of Science and at the time the

youngest elected vice-rector for science, education, and scientific-research work of the University of Sarajevo, as well as the president of the *Mathematics Section of the Association of Mathematicians, Physicists and Astronomers of Bosnia and Herzegovina*.

She received numerous awards and acknowledgements for her successful scientific work as well as contribution to the development of the University. We will only mention the highest ones: Republic Award *Veselin Masleša* (1987) for scientific work in the field of mathematics; the *Memorial Plaque of the City of Sarajevo* on the occasion of the 40th anniversary of liberation (1985); the *Order of Labor with a Silver Wreath* of the presidency of ex-Yugoslavia (1987); the *Charter on the occasion of the 50th anniversary of the founding of the University of Sarajevo* (1999). But she never leaves out her first awards: the silver and gold badges of the *Hasan Brkić University Fund* for excellent results and passing all her exams on time.

In 2012, she was elected a corresponding member, and in 2018, a full member of the Academy of Sciences and Arts of Bosnia and Herzegovina which represents the crown of her life's work.

Academician *Fikret Vajzović* wrote in the report for Mirjana Vuković's election to ANUBiH: *Mirjana Vuković is one of the most successful and educated mathematicians in Bosnia and Herzegovina. It is safe to say that she is among the scientists of a broad, not only mathematical, but also general education and culture* [...] *with the famous French mathematician Marc Krasner, she introduced and laid the foundations for a new and abstract theory – the theory of paragraded structures (groups, rings, modules) which is a generalization of the corresponding Bourbaki-Krasner graded structures and is already dubbed Krasner-Vuković paragraded structures after its authors. With those pioneering works, they signed their names as creators of a new theory, which is neither easy nor often in mathematics*.

Finaly, Mirjana was the first woman to win a Gold medal of Sarajevo University for the best students in mathematics, the second woman to defend her doctorate in mathematics at the University of Sarajevo, and the first woman ever elected to the Department of Natural Sciences and Mathematics of the Academy of Sciences and Arts of Bosnia and Herzegovina.

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