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ON STRONGLY REGULAR GRAPHS WITH $m_2 = qm_3$ AND $m_3 = qm_2$ FOR $q = 5,6,7,8$

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Dedicated to my mother Jordana Lepović

ABSTRACT. We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j, and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j, where S_k denotes the neighborhood of the vertex k. Let $\lambda_1 = r$, λ_2 and λ_3 be the distinct eigenvalues of a connected strongly regular graph. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r, λ_2 and λ_3, respectively. We describe the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5,6,7,8$.

1. INTRODUCTION

Let G be a simple graph of order n with vertex set $V(G) = \{1,2,\ldots,n\}$. The spectrum of G consists of the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ of its (0,1) adjacency matrix A and is denoted by $\sigma(G)$. We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph K_n) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j, and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j, where $S_k \subseteq V(G)$ denotes the neighborhood of the vertex k. We know that a regular connected graph G is strongly regular if and only if it has exactly three distinct eigenvalues [1] (see also [3]). Let $\lambda_1 = r$, λ_2 and λ_3 denote the distinct eigenvalues of a connected strongly regular graph G. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r, λ_2 and λ_3. Further, let $\overline{r} = (n-1)-r$, $\overline{\lambda}_2 = -\lambda_3 - 1$ and $\overline{\lambda}_3 = -\lambda_2 - 1$ denote the distinct eigenvalues of the strongly regular graph $\overline{G}$, where $\overline{G}$ denotes the complement of G. Then $\overline{\tau} = n - 2r - 2 + \theta$ and $\overline{\theta} = n - 2r + \tau$, where $\overline{\tau} = \tau(\overline{G})$ and $\overline{\theta} = \theta(\overline{G})$.

Remark 1.1. (i) if G is a disconnected strongly regular graph of degree r then $G = mK_{r+1}$, where mH denotes the m-fold union of the graph H; (ii) G is a disconnected strongly regular graph if and only if $\theta = 0$.

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Remark 1.2. (i) a strongly regular graph G of order $n = 4k+1$ and degree $r = 2k$ with $\tau = k-1$ and $\theta = k$ is called a conference graph; (ii) a strongly regular graph is a conference graph if and only if $m_2 = m_3$ and (iii) if $m_2 \neq m_3$ then G is an integral[1] graph.

We have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive integer [4]. In particular, in the same work we have described the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2,3,4$. We now proceed to establish the parameters of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5,6,7,8$, as follows. Firstly,

Proposition 1.1 (Elzinga [2])**.** *Let G be a connected or disconnected strongly regular graph of order n and degree r. Then*

$$r^2 - (\tau - \theta + 1)r - (n-1)\theta = 0\,. \tag{1.1}$$

Proposition 1.2 (Elzinga [2])**.** *Let G be a connected strongly regular graph of order n and degree r. Then*

$$2r + (\tau - \theta)(m_2 + m_3) + \delta(m_2 - m_3) = 0\,, \tag{1.2}$$

where $\delta = \lambda_2 - \lambda_3$.

Remark 1.3 (Lepović [4]). Using the same procedure applied in [4] we can establish the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for any fixed value $q \in \mathbb{N}$, as follows. Firstly, let $m_3 = p$, $m_2 = qp$ and $n = (q+1)p+1$, where $q \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - q\lambda_2)$. Let $|\lambda_3| - q\lambda_2 = t$, where $t = 1,2,\dots,q$. Let $\lambda_2 = k$, where k is a positive integer. Then (i) $\lambda_3 = -(qk+t)$; (ii) $\tau - \theta = -((q-1)k+t)$; (iii) $\delta = (q+1)k+t$; (iv) $r = pt$ and (v) $\theta = pt - qk^2 - kt$. Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(p+1)t^2 - \big((q+1)p+1\big)t + q(q+1)k^2 + 2qkt = 0\,. \tag{1.3}$$

Secondly, let $m_2 = p$, $m_3 = qp$ and $n = (q+1)p+1$, where $q \in \mathbb{N}$. Using (1.2) we obtain $r = p(q|\lambda_3| - \lambda_2)$. Let $q|\lambda_3| - \lambda_2 = t$, where $t = 1,2,\dots,q$. Let $\lambda_3 = -k$, where k is a positive integer. Then (i) $\lambda_2 = qk - t$; (ii) $\tau - \theta = (q-1)k - t$; (iii) $\delta = (q+1)k - t$; (iv) $r = pt$ and (v) $\theta = pt - qk^2 + kt$. Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(p+1)t^2 - \big((q+1)p+1\big)t + q(q+1)k^2 - 2qkt = 0\,. \tag{1.4}$$

Using (1.3) and (1.4) we can obtain for $t = 1,2,\dots,q$ the corresponding classes of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, respectively.

[1] We say that a connected or disconnected graph G is integral if its spectrum $\sigma(G)$ consists only of integral values.

2. MAIN RESULTS

Remark 2.1. We firstly describe the corresponding classes of connected strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ obtained in Propositions $2k+1$ and $2k+2$, respectively, then we prove Theorem k which is related to connected strongly regular graphs with $m_2 = qm_3$ or $m_3 = qm_2$, where $q = 5,6,7,8$ and $k = 1,2,3,4$.

Remark 2.2. Since $m_2(\overline{G}) = m_3(G)$ and $m_3(\overline{G}) = m_2(G)$ we note that if $m_2(G) = qm_3(G)$ then $m_3(\overline{G}) = qm_2(\overline{G})$.

Remark 2.3. In Theorems 2.1, 2.2, 2.3 and 2.4 the complements of strongly regular graphs appear in pairs in (k^0) and $(\overline{k}^0)$ classes, where k denotes the corresponding number of a class.

Remark 2.4. $\overline{\alpha K_\beta}$ is a strongly regular graph of order $n = \alpha\beta$ and degree $r = (\alpha-1)\beta$ with $\tau = (\alpha-2)\beta$ and $\theta = (\alpha-1)\beta$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -\beta$ with $m_2 = \alpha(\beta-1)$ and $m_3 = \alpha-1$.

Proposition 2.1. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 5m_3$. Then G belongs to the class $(\overline{2}^0)$ or (3^0) represented in Theorem* 2.1.

Proof. Let $m_3 = p$, $m_2 = 5p$ and $n = 6p+1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - 5\lambda_2)$. Let $|\lambda_3| - 5\lambda_2 = t$, where $t = 1,2,\ldots,5$. Let $\lambda_2 = k$, where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_3 = -(5k+t)$; (ii) $\tau - \theta = -(4k+t)$; (iii) $\delta = 6k+t$; (iv) $r = pt$ and (v) $\theta = pt - 5k^2 - kt$. In this case we can easily see that (1.3) is reduced to

$$(p+1)t^2 - (6p+1)t + 30k^2 + 10kt = 0\,. \tag{2.1}$$

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k+1)$, $\tau - \theta = -(4k+1)$, $\delta = 6k+1$, $r = p$ and $\theta = p - 5k^2 - k$. Using (2.1) we find that $p = 2k(3k+1)$. So we obtain that G is a strongly regular graph of order $n = (6k+1)^2$ and degree $r = 2k(3k+1)$ with $\tau = k^2 - 3k - 1$ and $\theta = k(k+1)$.
Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k+2)$, $\tau - \theta = -(4k+2)$, $\delta = 6k+2$, $r = 2p$ and $\theta = 2p - 5k^2 - 2k$. Using (2.1) we find that $4p - 1 = 5k(3k+2)$, a contradiction because $4 \nmid 15k^2 + 10k + 1$.
Case 3. ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k+3)$, $\tau - \theta = -(4k+3)$, $\delta = 6k+3$, $r = 3p$ and $\theta = 3p - 5k^2 - 3k$. Using (2.1) we find that $3p - 2 = 10k(k+1)$, a contradiction because $3 \nmid 10k^2 + 10k + 2$.
Case 4. ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k+4)$, $\tau - \theta = -(4k+4)$, $\delta = 6k+4$, $r = 4p$ and $\theta = 4p - 5k^2 - 4k$. Using (2.1) we find that $4p - 6 = 5k(3k+4)$, a contradiction because $4 \nmid 15k^2 + 20k + 6$.
Case 5. ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k+5)$, $\tau - \theta = -(4k+5)$, $\delta = 6k+5$, $r = 5p$ and $\theta = 5p - 5k^2 - 5k$. Using

(2.1) we find that $p = 2(k+1)(3k+2)$. Replacing k with $k-1$ we arrive at $p = 2k(3k-1)$. So we obtain that G is a strongly regular graph of order $n = (6k-1)^2$ and degree $r = 10k(3k-1)$ with $\tau = 25k^2 - 9k - 1$ and $\theta = 5k(5k-1)$. □

Proposition 2.2. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = 5m_2$. Then G belongs to the class (2^0) or $(\overline{3}^0)$ represented in Theorem* 2.1.

Proof. Let $m_2 = p$, $m_3 = 5p$ and $n = 6p+1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(5|\lambda_3| - \lambda_2)$. Let $5|\lambda_3| - \lambda_2 = t$, where $t = 1,2,\ldots,5$. Let $\lambda_3 = -k$, where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2 = 5k - t$; (ii) $\tau - \theta = 4k - t$; (iii) $\delta = 6k - t$; (iv) $r = pt$ and (v) $\theta = pt - 5k^2 + kt$. In this case we can easily see that (1.4) is reduced to

$$(p+1)t^2 - (6p+1)t + 30k^2 - 10kt = 0\,. \tag{2.2}$$

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 5k-1$ and $\lambda_3 = -k$, $\tau - \theta = 4k-1$, $\delta = 6k-1$, $r = p$ and $\theta = p - 5k^2 + k$. Using (2.2) we find that $p = 2k(3k-1)$. So we obtain that G is a strongly regular graph of order $n = (6k-1)^2$ and degree $r = 2k(3k-1)$ with $\tau = k^2 + 3k - 1$ and $\theta = k(k-1)$.
Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 5k-2$ and $\lambda_3 = -k$, $\tau - \theta = 4k-2$, $\delta = 6k-2$, $r = 2p$ and $\theta = 2p - 5k^2 + 2k$. Using (2.2) we find that $4p - 1 = 5k(3k-2)$, a contradiction because $4 \nmid 15k^2 - 10k + 1$.
Case 3. ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 5k-3$ and $\lambda_3 = -k$, $\tau - \theta = 4k-3$, $\delta = 6k-3$, $r = 3p$ and $\theta = 3p - 5k^2 + 3k$. Using (2.2) we find that $3p - 2 = 10k(k-1)$, a contradiction because $3 \nmid 10k^2 - 10k + 2$.
Case 4. ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 5k-4$ and $\lambda_3 = -k$, $\tau - \theta = 4k-4$, $\delta = 6k-4$, $r = 4p$ and $\theta = 4p - 5k^2 + 4k$. Using (2.2) we find that $4p - 6 = 5k(3k-4)$, a contradiction because $4 \nmid 15k^2 - 20k + 6$.
Case 5. ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 5k-5$ and $\lambda_3 = -k$, $\tau - \theta = 4k-5$, $\delta = 6k-5$, $r = 5p$ and $\theta = 5p - 5k^2 + 5k$. Using (2.2) we find that $p = 2(k-1)(3k-2)$. Replacing k with $k+1$ we arrive at $p = 2k((3k+1)$. So we obtain that G is a strongly regular graph of order $n = (6k+1)^2$ and degree $r = 10k(3k+1)$ with $\tau = 25k^2 + 9k - 1$ and $\theta = 5k(5k+1)$. □

Remark 2.5. We note that $\overline{5K_5}$ is a strongly regular graph with $m_2 = 5m_3$. It is obtained from the class Theorem 2.1 $(\overline{2}^0)$ for $k = 1$.

Theorem 2.1. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 5m_3$ or $m_3 = 5m_2$. Then G is one of the following strongly regular graphs:*

- (1^0) *G is the strongly regular graph $\overline{5K_5}$ of order $n = 25$ and degree $r = 20$ with $\tau = 15$ and $\theta = 20$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 20$ and $m_3 = 4$;*

(2^0) *G is a strongly regular graph of order $n = (6k-1)^2$ and degree $r = 2k(3k-1)$ with $\tau = k^2+3k-1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 5k-1$ and $\lambda_3 = -k$ with $m_2 = 2k(3k-1)$ and $m_3 = 10k(3k-1)$;*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (6k-1)^2$ and degree $r = 10k(3k-1)$ with $\tau = 25k^2-9k-1$ and $\theta = 5k(5k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k-1$ and $\lambda_3 = -5k$ with $m_2 = 10k(3k-1)$ and $m_3 = 2k(3k-1)$;*

(3^0) *G is a strongly regular graph of order $n = (6k+1)^2$ and degree $r = 2k(3k+1)$ with $\tau = k^2-3k-1$ and $\theta = k(k+1)$, where $k \geq 4$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(5k+1)$ with $m_2 = 10k(3k+1)$ and $m_3 = 2k(3k+1)$;*

($\overline{3}^0$) *G is a strongly regular graph of order $n = (6k+1)^2$ and degree $r = 10k(3k+1)$ with $\tau = 25k^2+9k-1$ and $\theta = 5k(5k+1)$, where $k \geq 4$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(k+1)$ with $m_2 = 2k(3k+1)$ and $m_3 = 10k(3k+1)$.*

Proof. Firstly, according to Remark 2.4 we have $\alpha(\beta-1) = 5(\alpha-1)$, from which we find that $\alpha = 5$, $\beta = 5$. In view of this we obtain the strongly regular graph represented in Theorem 2.1 (1^0). Next, according to Proposition 2.1 it turns out that G belongs to the class ($\overline{2}^0$) or (3^0) if $m_2 = 5m_3$. According to Proposition 2.2 it turns out that G belongs to the class (2^0) or ($\overline{3}^0$) if $m_3 = 5m_2$. □

Proposition 2.3. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 6m_3$. Then G belongs to the class ($\overline{4}^0$) or (5^0) or (6^0) or ($\overline{7}^0$) or (8^0) or ($\overline{9}^0$) represented in Theorem 2.2.*

Proof. Let $m_3 = p$, $m_2 = 6p$ and $n = 7p+1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - 6\lambda_2)$. Let $|\lambda_3| - 6\lambda_2 = t$, where $t = 1,2,\ldots,6$. Let $\lambda_2 = k$, where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_3 = -(6k+t)$; (ii) $\tau - \theta = -(5k+t)$; (iii) $\delta = 7k+t$; (iv) $r = pt$ and (v) $\theta = pt - 6k^2 - kt$. In this case we can easily see that (1.3) is reduced to

$$(p+1)t^2 - (7p+1)t + 42k^2 + 12kt = 0. \tag{2.3}$$

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(6k+1)$, $\tau - \theta = -(5k+1)$, $\delta = 7k+1$, $r = p$ and $\theta = p - 6k^2 - k$. Using (2.3) we find that $p = k(7k+2)$. So we obtain that G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = k(7k+2)$ with $\tau = k^2 - 4k - 1$ and $\theta = k(k+1)$.

Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(6k+2)$, $\tau - \theta = -(5k+2)$, $\delta = 7k+2$, $r = 2p$ and $\theta = 2p - 6k^2 - 2k$. Using (2.3) we find that $5p - 1 = 3k(7k+4)$. Replacing k with $5k-1$ we arrive at $p = 105k^2 - 30k + 2$. So we obtain that G is a strongly regular graph of order $n = 15(7k-1)^2$ and degree $r = 2(105k^2 - 30k + 2)$ with $\tau = 60k^2 - 35k + 3$ and $\theta = 10k(6k-1)$.

Case 3. ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(6k+3)$, $\tau - \theta = -(5k+3)$, $\delta = 7k+3$, $r = 3p$ and $\theta = 3p - 6k^2 - 3k$. Using

(2.3) we find that $2p-1=k(7k+6)$. Replacing k with $2k-1$ we arrive at $p=14k^2-8k+1$. So we obtain that G is a strongly regular graph of order $n=2(7k-2)^2$ and degree $r=3(14k^2-8k+1)$ with $\tau=18k^2-16k+2$ and $\theta=6k(3k-1)$.
Case 4. $(t=4)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(6k+4)$, $\tau-\theta=-(5k+4)$, $\delta=7k+4$, $r=4p$ and $\theta=4p-6k^2-4k$. Using (2.3) we find that $2p-2=k(7k+8)$. Replacing k with $2k$ we arrive at $p=14k^2+8k+1$. So we obtain that G is a strongly regular graph of order $n=2(7k+2)^2$ and degree $r=4(14k^2+8k+1)$ with $\tau=2k(16k+7)$ and $\theta=4(2k+1)(4k+1)$.
Case 5. $(t=5)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(6k+5)$, $\tau-\theta=-(5k+5)$, $\delta=7k+5$, $r=5p$ and $\theta=5p-6k^2-5k$. Using (2.3) we find that $5p-10=3k(7k+10)$. Replacing k with $5k$ we arrive at $p=105k^2+30k+2$. So we obtain that G is a strongly regular graph of order $n=15(7k+1)^2$ and degree $r=5(105k^2+30k+2)$ with $\tau=5(5k+1)(15k+1)$ and $\theta=5(5k+1)(15+2)$.
Case 6. $(t=6)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(6k+6)$, $\tau-\theta=-(5k+6)$, $\delta=7k+6$, $r=6p$ and $\theta=6p-6k^2-6k$. Using (2.3) we find that $p=(k+1)(7k+5)$. Replacing k with $k-1$ we arrive at $p=k(7k-2)$. So we obtain that G is a strongly regular graph of order $n=(7k-1)^2$ and degree $r=6k(7k-2)$ with $\tau=36k^2-11k-1$ and $\theta=6k(6k-1)$. □

Proposition 2.4. *Let G be a connected strongly regular graph of order n and degree r with $m_3=6m_2$. Then G belongs to the class* (4^0) *or* $(\overline{5}^0)$ *or* $(\overline{6}^0)$ *or* (7^0) *or* $(\overline{8}^0)$ *or* (9^0) *represented in Theorem* 2.2.

Proof. Let $m_2=p$, $m_3=6p$ and $n=7p+1$, where $p\in\mathbb{N}$. Using (1.2) we obtain $r=p(6|\lambda_3|-\lambda_2)$. Let $6|\lambda_3|-\lambda_2=t$, where $t=1,2,\ldots,6$. Let $\lambda_3=-k$, where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2=6k-t$; (ii) $\tau-\theta=5k-t$; (iii) $\delta=7k-t$; (iv) $r=pt$ and (v) $\theta=pt-6k^2+kt$. In this case we can easily see that (1.4) is reduced to

$$(p+1)t^2-(7p+1)t+42k^2-12kt=0\,. \tag{2.4}$$

Case 1. $(t=1)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=6k-1$ and $\lambda_3=-k$, $\tau-\theta=5k-1$, $\delta=7k-1$, $r=p$ and $\theta=p-6k^2+k$. Using (2.4) we find that $p=k(7k-2)$. So we obtain that G is a strongly regular graph of order $n=(7k-1)^2$ and degree $r=k(7k-2)$ with $\tau=k^2+4k-1$ and $\theta=k(k-1)$.
Case 2. $(t=2)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=6k-2$ and $\lambda_3=-k$, $\tau-\theta=5k-2$, $\delta=7k-2$, $r=2p$ and $\theta=2p-6k^2+2k$. Using (2.4) we find that $5p-1=3k(7k-4)$. Replacing k with $5k+1$ we arrive at $p=105k^2+30k+2$. So we obtain that G is a strongly regular graph of order $n=15(7k+1)^2$ and degree $r=2(105k^2+30k+2)$ with $\tau=60k^2+35k+3$ and $\theta=10k(6k+1)$.
Case 3. $(t=3)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=6k-3$ and $\lambda_3=-k$, $\tau-\theta=5k-3$, $\delta=7k-3$, $r=3p$ and $\theta=3p-6k^2+3k$. Using (2.4) we

find that $2p-1 = k(7k-6)$. Replacing k with $2k+1$ we arrive at $p = 14k^2+8k+1$. So we obtain that G is a strongly regular graph of order $n = 2(7k+2)^2$ and degree $r = 3(14k^2+8k+1)$ with $\tau = 18k^2+16k+2$ and $\theta = 6k(3k+1)$.
Case 4. ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k-4$ and $\lambda_3 = -k$, $\tau-\theta = 5k-4$, $\delta = 7k-4$, $r = 4p$ and $\theta = 4p-6k^2+4k$. Using (2.4) we find that $2p-2 = k(7k-8)$. Replacing k with $2k$ we arrive at $p = 14k^2-8k+1$. So we obtain that G is a strongly regular graph of order $n = 2(7k-2)^2$ and degree $r = 4(14k^2-8k+1)$ with $\tau = 2k(16k-7)$ and $\theta = 4(2k-1)(4k-1)$.
Case 5. ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k-5$ and $\lambda_3 = -k$, $\tau-\theta = 5k-5$, $\delta = 7k-5$, $r = 5p$ and $\theta = 5p-6k^2+5k$. Using (2.4) we find that $5p-10 = 3k(7k-10)$. Replacing k with $5k$ we arrive at $p = 105k^2-30k+2$. So we obtain that G is a strongly regular graph of order $n = 15(7k-1)^2$ and degree $r = 5(105k^2-30k+2)$ with $\tau = 5(5k-1)(15k-1)$ and $\theta = 5(5k-1)(15k-2)$.
Case 6. ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k-6$ and $\lambda_3 = -k$, $\tau-\theta = 5k-6$, $\delta = 7k-6$, $r = 6p$ and $\theta = 6p-6k^2+6k$. Using (2.4) we find that $p = (k-1)(7k-5)$. Replacing k with $k+1$ we arrive at $p = k(7k+2)$. So we obtain that G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 6k(7k+2)$ with $\tau = 36k^2+11k-1$ and $\theta = 6k(6k+1)$. □

Remark 2.6. We note that the complete bipartite graph $K_{4,4}$ is a strongly regular graph with $m_2 = 6m_3$. It is obtained from the class Theorem 2.2 $(\overline{7}^0)$ for $k = 0$.

Remark 2.7. We note that $\overline{3K_5}$ is a strongly regular graph with $m_2 = 6m_3$. It is obtained from the class Theorem 2.2 $(\overline{9}^0)$ for $k = 0$.

Remark 2.8. We note that $\overline{6K_6}$ is a strongly regular graph with $m_2 = 6m_3$. It is obtained from the class Theorem 2.2 $(\overline{4}^0)$ for $k = 1$.

Theorem 2.2. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 6m_3$ or $m_3 = 6m_2$. Then G is one of the following strongly regular graphs:*

- (1^0) *G is the complete bipartite graph $K_{4,4}$ of order $n = 8$ and degree $r = 4$ with $\tau = 0$ and $\theta = 4$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -4$ with $m_2 = 6$ and $m_3 = 1$;*
- (2^0) *G is the strongly regular graph $\overline{3K_5}$ of order $n = 15$ and degree $r = 10$ with $\tau = 5$ and $\theta = 10$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 12$ and $m_3 = 2$;*
- (3^0) *G is the strongly regular graph $\overline{6K_6}$ of order $n = 36$ and degree $r = 30$ with $\tau = 24$ and $\theta = 30$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -6$ with $m_2 = 30$ and $m_3 = 5$;*
- (4^0) *G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = k(7k-2)$ with $\tau = k^2+4k-1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 6k-1$ and $\lambda_3 = -k$ with $m_2 = k(7k-2)$ and $m_3 = 6k(7k-2)$;*

($\overline{4}^0$) *G is a strongly regular graph of order* $n=(7k-1)^2$ *and degree* $r=6k(7k-2)$ *with* $\tau=36k^2-11k-1$ *and* $\theta=6k(6k-1)$*, where* $k\geq 2$*. Its eigenvalues are* $\lambda_2=k-1$ *and* $\lambda_3=-6k$ *with* $m_2=6k(7k-2)$ *and* $m_3=k(7k-2)$ *;*

(5^0) *G is a strongly regular graph of order* $n=(7k+1)^2$ *and degree* $r=k(7k+2)$ *with* $\tau=k^2-4k-1$ *and* $\theta=k(k+1)$*, where* $k\geq 5$*. Its eigenvalues are* $\lambda_2=k$ *and* $\lambda_3=-(6k+1)$ *with* $m_2=6k(7k+2)$ *and* $m_3=k(7k+2)$ *;*

($\overline{5}^0$) *G is a strongly regular graph of order* $n=(7k+1)^2$ *and degree* $r=6k(7k+2)$ *with* $\tau=36k^2+11k-1$ *and* $\theta=6k(6k+1)$*, where* $k\geq 5$*. Its eigenvalues are* $\lambda_2=6k$ *and* $\lambda_3=-(k+1)$ *with* $m_2=k(7k+2)$ *and* $m_3=6k(7k+2)$ *;*

(6^0) *G is a strongly regular graph of order* $n=2(7k-2)^2$ *and degree* $r=3(14k^2-8k+1)$ *with* $\tau=18k^2-16k+2$ *and* $\theta=6k(3k-1)$*, where* $k\in\mathbb{N}$*. Its eigenvalues are* $\lambda_2=2k-1$ *and* $\lambda_3=-(12k-3)$ *with* $m_2=6(14k^2-8k+1)$ *and* $m_3=14k^2-8k+1$ *;*

($\overline{6}^0$) *G is a strongly regular graph of order* $n=2(7k-2)^2$ *and degree* $r=4(14k^2-8k+1)$ *with* $\tau=2k(16k-7)$ *and* $\theta=4(2k-1)(4k-1)$*, where* $k\in\mathbb{N}$*. Its eigenvalues are* $\lambda_2=12k-4$ *and* $\lambda_3=-2k$ *with* $m_2=14k^2-8k+1$ *and* $m_3=6(14k^2-8k+1)$ *;*

(7^0) *G is a strongly regular graph of order* $n=2(7k+2)^2$ *and degree* $r=3(14k^2+8k+1)$ *with* $\tau=18k^2+16k+2$ *and* $\theta=6k(3k+1)$*, where* $k\in\mathbb{N}$*. Its eigenvalues are* $\lambda_2=12k+3$ *and* $\lambda_3=-(2k+1)$ *with* $m_2=14k^2+8k+1$ *and* $m_3=6(14k^2+8k+1)$ *;*

($\overline{7}^0$) *G is a strongly regular graph of order* $n=2(7k+2)^2$ *and degree* $r=4(14k^2+8k+1)$ *with* $\tau=2k(16k+7)$ *and* $\theta=4(2k+1)(4k+1)$*, where* $k\in\mathbb{N}$*. Its eigenvalues are* $\lambda_2=2k$ *and* $\lambda_3=-(12k+4)$ *with* $m_2=6(14k^2+8k+1)$ *and* $m_3=14k^2+8k+1$ *;*

(8^0) *G is a strongly regular graph of order* $n=15(7k-1)^2$ *and degree* $r=2(105k^2-30k+2)$ *with* $\tau=60k^2-35k+3$ *and* $\theta=10k(6k-1)$*, where* $k\in\mathbb{N}$*. Its eigenvalues are* $\lambda_2=5k-1$ *and* $\lambda_3=-(30k-4)$ *with* $m_2=6(105k^2-30k+2)$ *and* $m_3=105k^2-30k+2$ *;*

($\overline{8}^0$) *G is a strongly regular graph of order* $n=15(7k-1)^2$ *and degree* $r=5(105k^2-30k+2)$ *with* $\tau=5(5k-1)(15k-1)$ *and* $\theta=5(5k-1)(15k-2)$*, where* $k\in\mathbb{N}$*. Its eigenvalues are* $\lambda_2=30k-5$ *and* $\lambda_3=-5k$ *with* $m_2=105k^2-30k+2$ *and* $m_3=6(105k^2-30k+2)$ *;*

(9^0) *G is a strongly regular graph of order* $n=15(7k+1)^2$ *and degree* $r=2(105k^2+30k+2)$ *with* $\tau=60k^2+35k+3$ *and* $\theta=10k(6k+1)$*, where* $k\in\mathbb{N}$*. Its eigenvalues are* $\lambda_2=30k+4$ *and* $\lambda_3=-(5k+1)$ *with* $m_2=105k^2+30k+2$ *and* $m_3=6(105k^2+30k+2)$ *;*

($\overline{9}^0$) *G is a strongly regular graph of order* $n=15(7k+1)^2$ *and degree* $r=5(105k^2+30k+2)$ *with* $\tau=5(5k+1)(15k+1)$ *and* $\theta=5(5k+1)(15+2)$*,* *where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(30k+5)$ with $m_2 = 6(105k^2+30k+2)$ and $m_3 = 105k^2+30k+2$;*

Proof. Firstly, according to Remark 2.4 we have $\alpha(\beta-1) = 6(\alpha-1)$, from which we find that $\alpha = 2$, $\beta = 4$ or $\alpha = 3$, $\beta = 5$ or $\alpha = 6$, $\beta = 6$. In view of this we obtain the strongly regular graphs represented in Theorem 2.2 (1^0), (2^0), (3^0). Next, according to Proposition 2.3 it turns out that G belongs to the class $(\overline{4}^0)$ or (5^0) or (6^0) or $(\overline{7}^0)$ or (8^0) or $(\overline{9}^0)$ if $m_2 = 6m_3$. According to Proposition 2.4 it turns out that G belongs to the class (4^0) or $(\overline{5}^0)$ or $(\overline{6}^0)$ or (7^0) or $(\overline{8}^0)$ or (9^0) if $m_3 = 6m_2$. □

Proposition 2.5. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 7m_3$. Then G belongs to the class $(\overline{2}^0)$ or (3^0) or (4^0) or $(\overline{5}^0)$ represented in Theorem* 2.3*.*

Proof. Let $m_3 = p$, $m_2 = 7p$ and $n = 8p+1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - 7\lambda_2)$. Let $|\lambda_3| - 7\lambda_2 = t$, where $t = 1,2,\ldots,7$. Let $\lambda_2 = k$, where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_3 = -(7k+t)$; (ii) $\tau - \theta = -(6k+t)$; (iii) $\delta = 8k+t$; (iv) $r = pt$ and (v) $\theta = pt - 7k^2 - kt$. In this case we can easily see that (1.3) is reduced to

$$(p+1)t^2 - (8p+1)t + 56k^2 + 14kt = 0\,. \tag{2.5}$$

Case 1. $(t = 1)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k+1)$, $\tau - \theta = -(6k+1)$, $\delta = 8k+1$, $r = p$ and $\theta = p - 7k^2 - k$. Using (2.5) we find that $p = 2k(4k+1)$. So we obtain that G is a strongly regular graph of order $n = (8k+1)^2$ and degree $r = 2k(4k+1)$ with $\tau = k^2 - 5k - 1$ and $\theta = k(k+1)$.
Case 2. $(t = 2)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k+2)$, $\tau - \theta = -(6k+2)$, $\delta = 8k+2$, $r = 2p$ and $\theta = 2p - 7k^2 - 2k$. Using (2.5) we find that $6p - 1 = 14k(2k+1)$, a contradiction because $2 \nmid 6p - 1$.
Case 3. $(t = 3)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k+3)$, $\tau - \theta = -(6k+3)$, $\delta = 8k+3$, $r = 3p$ and $\theta = 3p - 7k^2 - 3k$. Using (2.5) we find that $15p - 6 = 14k(4k+3)$. Replacing k with $15k - 6$ we arrive at $p = 840k^2 - 630k + 118$. So we obtain that G is a strongly regular graph of order $n = 105(8k-3)^2$ and degree $r = 3(840k^2 - 630k + 118)$ with $\tau = 945k^2 - 765k + 153$ and $\theta = 15(3k-1)(21k-8)$.
Case 4. $(t = 4)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k+4)$, $\tau - \theta = -(6k+4)$, $\delta = 8k+4$, $r = 4p$ and $\theta = 4p - 7k^2 - 4k$. Using (2.5) we find that $4p - 3 = 14k(k+1)$, a contradiction because $2 \nmid 4p - 3$.
Case 5. $(t = 5)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k+5)$, $\tau - \theta = -(6k+5)$, $\delta = 8k+5$, $r = 5p$ and $\theta = 5p - 7k^2 - 5k$. Using (2.5) we find that $15p - 20 = 14k(4k+5)$. Replacing k with $15k + 5$ we arrive at $p = 840k^2 + 630k + 118$. So we obtain that G is a strongly regular graph of order

$n = 105(8k+3)^2$ and degree $r = 5(840k^2+630k+118)$ with $\tau = 5(525k^2+387k+71)$ and $\theta = 15(5k+2)(35k+13)$.
Case 6. ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k+6)$, $\tau-\theta = -(6k+6)$, $\delta = 8k+6$, $r = 6p$ and $\theta = 6p-7k^2-6k$. Using (2.5) we find that $6p-15 = 14k(2k+3)$, a contradiction because $2 \nmid 6p-15$.
Case 7. ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k+7)$, $\tau-\theta = -(6k+7)$, $\delta = 8k+7$, $r = 7p$ and $\theta = 7p-7k^2-7k$. Using (2.5) we find that $p = 2(k+1)(4k+3)$. Replacing k with $k-1$ we arrive at $p = 2k(4k-1)$. So we obtain that G is a strongly regular graph of order $n = (8k-1)^2$ and degree $r = 14k(4k-1)$ with $\tau = 49k^2-13k-1$ and $\theta = 7k(7k-1)$. $\square$

Proposition 2.6. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = 7m_2$. Then G belongs to the class* (2^0) *or* $(\overline{3}^0)$ *or* $(\overline{4}^0)$ *or* (5^0) *represented in Theorem* 2.3*.*

Proof. Let $m_2 = p$, $m_3 = 7p$ and $n = 8p+1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(7|\lambda_3|-\lambda_2)$. Let $7|\lambda_3|-\lambda_2 = t$, where $t = 1,2,\dots,7$. Let $\lambda_3 = -k$, where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2 = 7k-t$; (ii) $\tau-\theta = 6k-t$; (iii) $\delta = 8k-t$; (iv) $r = pt$ and (v) $\theta = pt-7k^2+kt$. In this case we can easily see that (1.4) is reduced to

$$(p+1)t^2-(8p+1)t+56k^2-14kt = 0. \tag{2.6}$$

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k-1$ and $\lambda_3 = -k$, $\tau-\theta = 6k-1$, $\delta = 8k-1$, $r = p$ and $\theta = p-7k^2+k$. Using (2.6) we find that $p = 2k(4k-1)$. So we obtain that G is a strongly regular graph of order $n = (8k-1)^2$ and degree $r = 2k(4k-1)$ with $\tau = k^2+5k-1$ and $\theta = k(k-1)$.
Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k-2$ and $\lambda_3 = -k$, $\tau-\theta = 6k-2$, $\delta = 8k-2$, $r = 2p$ and $\theta = 2p-7k^2+2k$. Using (2.6) we find that $6p-1 = 14k(2k-1)$, a contradiction because $2 \nmid 6p-1$.
Case 3. ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k-3$ and $\lambda_3 = -k$, $\tau-\theta = 6k-3$, $\delta = 8k-3$, $r = 3p$ and $\theta = 3p-7k^2+3k$. Using (2.6) we find that $15p-6 = 14k(4k-3)$. Replacing k with $15k+6$ we arrive at $p = 840k^2+630k+118$. So we obtain that G is a strongly regular graph of order $n = 105(8k+3)^2$ and degree $r = 3(840k^2+630k+118)$ with $\tau = 945k^2+765k+153$ and $\theta = 15(3k+1)(21k+8)$.
Case 4. ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k-4$ and $\lambda_3 = -k$, $\tau-\theta = 6k-4$, $\delta = 8k-4$, $r = 4p$ and $\theta = 4p-7k^2+4k$. Using (2.6) we find that $4p-3 = 14k(k-1)$, a contradiction because $2 \nmid 4p-3$.
Case 5. ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k-5$ and $\lambda_3 = -k$, $\tau-\theta = 6k-5$, $\delta = 8k-5$, $r = 5p$ and $\theta = 5p-7k^2+5k$. Using (2.6) we find that $15p-20 = 14k(4k-5)$. Replacing k with $15k-5$ we arrive at $p = 840k^2-630k+118$. So we obtain that G is a strongly regular graph of order $n =$

$105(8k-3)^2$ and degree $r = 5(840k^2-630k+118)$ with $\tau = 5(525k^2-387k+71)$ and $\theta = 15(5k-2)(35k-13)$.

Case 6. ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k-6$ and $\lambda_3 = -k$, $\tau-\theta = 6k-6$, $\delta = 8k-6$, $r = 6p$ and $\theta = 6p-7k^2+6k$. Using (2.6) we find that $6p-15 = 14k(2k-3)$, a contradiction because $2 \nmid 6p-15$.

Case 7. ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k-7$ and $\lambda_3 = -k$, $\tau-\theta = 6k-7$, $\delta = 8k-7$, $r = 7p$ and $\theta = 7p-7k^2+7k$. Using (2.6) we find that $p = 2(k-1)(4k-3)$. Replacing k with $k+1$ we arrive at $p = 2k(4k+1)$. So we obtain that G is a strongly regular graph of order $n = (8k+1)^2$ and degree $r = 14k(4k+1)$ with $\tau = 49k^2+13k-1$ and $\theta = 7k(7k+1)$. □

Remark 2.9. We note that $\overline{7K_7}$ is a strongly regular graph with $m_2 = 7m_3$. It is obtained from the class Theorem 2.3 ($\overline{2}^0$) for $k = 1$.

Theorem 2.3. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 7m_3$ or $m_3 = 7m_2$. Then G is one of the following strongly regular graphs:*

- (1^0) *G is the strongly regular graph $\overline{7K_7}$ of order $n = 49$ and degree $r = 42$ with $\tau = 35$ and $\theta = 42$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 42$ and $m_3 = 6$;*
- (2^0) *G is a strongly regular graph of order $n = (8k-1)^2$ and degree $r = 2k(4k-1)$ with $\tau = k^2+5k-1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 7k-1$ and $\lambda_3 = -k$ with $m_2 = 2k(4k-1)$ and $m_3 = 14k(4k-1)$;*
- ($\overline{2}^0$) *G is a strongly regular graph of order $n = (8k-1)^2$ and degree $r = 14k(4k-1)$ with $\tau = 49k^2-13k-1$ and $\theta = 7k(7k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k-1$ and $\lambda_3 = -7k$ with $m_2 = 14k(4k-1)$ and $m_3 = 2k(4k-1)$;*
- (3^0) *G is a strongly regular graph of order $n = (8k+1)^2$ and degree $r = 2k(4k+1)$ with $\tau = k^2-5k-1$ and $\theta = k(k+1)$, where $k \geq 6$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(7k+1)$ with $m_2 = 14k(4k+1)$ and $m_3 = 2k(4k+1)$;*
- ($\overline{3}^0$) *G is a strongly regular graph of order $n = (8k+1)^2$ and degree $r = 14k(4k+1)$ with $\tau = 49k^2+13k-1$ and $\theta = 7k(7k+1)$, where $k \geq 6$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(k+1)$ with $m_2 = 2k(4k+1)$ and $m_3 = 14k(4k+1)$;*
- (4^0) *G is a strongly regular graph of order $n = 105(8k-3)^2$ and degree $r = 3(840k^2-630k+118)$ with $\tau = 945k^2-765k+153$ and $\theta = 15(3k-1)(21k-8)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 15k-6$ and $\lambda_3 = -(105k-39)$ with $m_2 = 7(840k^2-630k+118)$ and $m_3 = 840k^2-630k+118$;*
- ($\overline{4}^0$) *G is a strongly regular graph of order $n = 105(8k-3)^2$ and degree $r = 5(840k^2-630k+118)$ with $\tau = 5(525k^2-387k+71)$ and $\theta = 15(5k-2)(35k-13)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 105k-40$ and $\lambda_3 = -(15k-5)$ with $m_2 = 840k^2-630k+118$ and $m_3 = 7(840k^2-630k+118)$;*
- (5^0) *G is a strongly regular graph of order $n = 105(8k+3)^2$ and degree $r = 3(840k^2+630k+118)$ with $\tau = 945k^2+765k+153$ and $\theta = 15(3k+1)(21k+$*

8), *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 105k+39$ *and* $\lambda_3 = -(15k+6)$ *with* $m_2 = 840k^2+630k+118$ *and* $m_3 = 7(840k^2+630k+118)$ *;*

$(\overline{5}^0)$ *G is a strongly regular graph of order* $n = 105(8k+3)^2$ *and degree* $r = 5(840k^2+630k+118)$ *with* $\tau = 5(525k^2+387k+71)$ *and* $\theta = 15(5k+2)(35k+13)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 15k+5$ *and* $\lambda_3 = -(105k+40)$ *with* $m_2 = 7(840k^2+630k+118)$ *and* $m_3 = 840k^2+630k+118$.

Proof. Firstly, according to Remark 2.4 we have $\alpha(\beta-1) = 7(\alpha-1)$, from which we find that $\alpha = 7$, $\beta = 7$. In view of this we obtain the strongly regular graph represented in Theorem 2.3 (1^0). Next, according to Proposition 2.5 it turns out that G belongs to the class $(\overline{2}^0)$ or (3^0) or (4^0) or $(\overline{5}^0)$ if $m_2 = 7m_3$. According to Proposition 2.6 it turns out that G belongs to the class (2^0) or $(\overline{3}^0)$ or $(\overline{4}^0)$ or (5^0) if $m_3 = 7m_2$. □

Proposition 2.7. *Let G be a connected strongly regular graph of order n and degree r with* $m_2 = 8m_3$. *Then G belongs to the class* $(\overline{4}^0)$ *or* (5^0) *or* $(\overline{6}^0)$ *or* (7^0) *or* (8^0) *or* $(\overline{9}^0)$ *or* (10^0) *or* $(\overline{11}^0)$ *represented in Theorem* 2.4.

Proof. Let $m_3 = p$, $m_2 = 8p$ and $n = 9p+1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - 8\lambda_2)$. Let $|\lambda_3| - 8\lambda_2 = t$, where $t = 1, 2, \ldots, 8$. Let $\lambda_2 = k$, where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_3 = -(8k+t)$; (ii) $\tau - \theta = -(7k+t)$; (iii) $\delta = 9k+t$; (iv) $r = pt$ and (v) $\theta = pt - 8k^2 - kt$. In this case we can easily see that (1.3) is reduced to

$$(p+1)t^2 - (9p+1)t + 72k^2 + 16kt = 0. \tag{2.7}$$

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k+1)$, $\tau - \theta = -(7k+1)$, $\delta = 9k+1$, $r = p$ and $\theta = p - 8k^2 - k$. Using (2.7) we find that $p = k(9k+2)$. So we obtain that G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = k(9k+2)$ with $\tau = k^2 - 6k - 1$ and $\theta = k(k+1)$.

Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k+2)$, $\tau - \theta = -(7k+2)$, $\delta = 9k+2$, $r = 2p$ and $\theta = 2p - 8k^2 - 2k$. Using (2.7) we find that $7p - 1 = 4k(9k+4)$. Replacing k with $7k-1$ we arrive at $p = 252k^2 - 56k + 3$. So we obtain that G is a strongly regular graph of order $n = 28(9k-1)^2$ and degree $r = 2(252k^2 - 56k + 3)$ with $\tau = 112k^2 - 63k + 5$ and $\theta = 14k(8k-1)$.

Case 3. ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k+3)$, $\tau - \theta = -(7k+3)$, $\delta = 9k+3$, $r = 3p$ and $\theta = 3p - 8k^2 - 3k$. Using (2.7) we find that $3p - 1 = 4k(3k+2)$. Replacing k with $3k+1$ we arrive at $p = (2k+1)(18k+7)$. So we obtain that G is a strongly regular graph of order $n = 4(9k+4)^2$ and degree $r = 3(2k+1)(18k+7)$ with $\tau = 18k(2k+1)$ and $\theta = (3k+2)(12k+5)$.

Case 4. ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k+4)$, $\tau - \theta = -(7k+4)$, $\delta = 9k+4$, $r = 4p$ and $\theta = 4p - 8k^2 - 4k$. Using

(2.7) we find that $5p-3 = 2k(9k+8)$. Replacing k with $5k-1$ we arrive at $p = 90k^2-20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k-1)^2$ and degree $r = 4(90k^2-20k+1)$ with $\tau = 160k^2-55k+3$ and $\theta = 20k(8k-1)$.
Case 5. ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k+5)$, $\tau-\theta = -(7k+5)$, $\delta = 9k+5$, $r = 5p$ and $\theta = 5p-8k^2-5k$. Using (2.7) we find that $5p-5 = 2k(9k+10)$. Replacing k with $5k$ we arrive at $p = 90k^2+20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k+1)^2$ and degree $r = 5(90k^2+20k+1)$ with $\tau = 10k(25k+4)$ and $\theta = 5(5k+1)(10k+1)$.
Case 6. ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k+6)$, $\tau-\theta = -(7k+6)$, $\delta = 9k+6$, $r = 6p$ and $\theta = 6p-8k^2-6k$. Using (2.7) we find that $3p-5 = 4k(3k+4)$. Replacing k with $3k-2$ we arrive at $p = (2k-1)(18k-7)$. So we obtain that G is a strongly regular graph of order $n = 4(9k-4)^2$ and degree $r = 6(2k-1)(18k-7)$ with $\tau = 3(48k^2-45k+10)$ and $\theta = 2(3k-1)(24k-11)$.
Case 7. ($t = 7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k+7)$, $\tau-\theta = -(7k+7)$, $\delta = 9k+7$, $r = 7p$ and $\theta = 7p-8k^2-7k$. Using (2.7) we find that $7p-21 = 4k(9k+14)$. Replacing k with $7k$ we arrive at $p = 252k^2+56k+3$. So we obtain that G is a strongly regular graph of order $n = 28(9k+1)^2$ and degree $r = 7(252k^2+56k+3)$ with $\tau = 14(7k+1)(14k+1)$ and $\theta = 7(7k+1)(28k+3)$.
Case 8. ($t = 8$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k+8)$, $\tau-\theta = -(7k+8)$, $\delta = 9k+8$, $r = 8p$ and $\theta = 8p-8k^2-8k$. Using (2.7) we find that $p = (k+1)(9k+7)$. Replacing k with $k-1$ we arrive at $p = k(9k-2)$. So we obtain that G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = 8k(9k-2)$ with $\tau = 64k^2-15k-1$ and $\theta = 8k(8k-1)$. □

Proposition 2.8. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = 8m_2$. Then G belongs to the class* (4^0) *or* $(\overline{5}^0)$ *or* (6^0) *or* $(\overline{7}^0)$ *or* $(\overline{8}^0)$ *or* (9^0) *or* $(\overline{10}^0)$ *or* (11^0) *represented in Theorem* 2.4.

Proof. Let $m_2 = p$, $m_3 = 8p$ and $n = 9p+1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(8|\lambda_3|-\lambda_2)$. Let $8|\lambda_3|-\lambda_2 = t$, where $t = 1,2,\ldots,8$. Let $\lambda_3 = -k$, where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2 = 8k-t$; (ii) $\tau-\theta = 7k-t$; (iii) $\delta = 9k-t$; (iv) $r = pt$ and (v) $\theta = pt-8k^2+kt$. In this case we can easily see that (1.4) is reduced to

$$(p+1)t^2-(9p+1)t+72k^2-16kt = 0. \tag{2.8}$$

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k-1$ and $\lambda_3 = -k$, $\tau-\theta = 7k-1$, $\delta = 9k-1$, $r = p$ and $\theta = p-8k^2+k$. Using (2.8) we find that $p = k(9k-2)$. So we obtain that G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = k(9k-2)$ with $\tau = k^2+6k-1$ and $\theta = k(k-1)$.

Case 2. ($t=2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k-2$ and $\lambda_3 = -k$, $\tau-\theta = 7k-2$, $\delta = 9k-2$, $r = 2p$ and $\theta = 2p-8k^2+2k$. Using (2.8) we find that $7p-1 = 4k(9k-4)$. Replacing k with $7k+1$ we arrive at $p = 252k^2+56k+3$. So we obtain that G is a strongly regular graph of order $n = 28(9k+1)^2$ and degree $r = 2(252k^2+56k+3)$ with $\tau = 112k^2+63k+5$ and $\theta = 14k(8k+1)$.
Case 3. ($t=3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k-3$ and $\lambda_3 = -k$, $\tau-\theta = 7k-3$, $\delta = 9k-3$, $r = 3p$ and $\theta = 3p-8k^2+3k$. Using (2.8) we find that $3p-1 = 4k(3k-2)$. Replacing k with $3k-1$ we arrive at $p = (2k-1)(18k-7)$. So we obtain that G is a strongly regular graph of order $n = 4(9k-4)^2$ and degree $r = 3(2k-1)(18k-7)$ with $\tau = 18k(2k-1)$ and $\theta = (3k-2)(12k-5)$.
Case 4. ($t=4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k-4$ and $\lambda_3 = -k$, $\tau-\theta = 7k-4$, $\delta = 9k-4$, $r = 4p$ and $\theta = 4p-8k^2+4k$. Using (2.8) we find that $5p-3 = 2k(9k-8)$. Replacing k with $5k+1$ we arrive at $p = 90k^2+20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k+1)^2$ and degree $r = 4(90k^2+20k+1)$ with $\tau = 160k^2+55k+3$ and $\theta = 20k(8k+1)$.
Case 5. ($t=5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k-5$ and $\lambda_3 = -k$, $\tau-\theta = 7k-5$, $\delta = 9k-5$, $r = 5p$ and $\theta = 5p-8k^2+5k$. Using (2.8) we find that $5p-5 = 2k(9k-10)$. Replacing k with $5k$ we arrive at $p = 90k^2-20k+1$. So we obtain that G is a strongly regular graph of order $n = 10(9k-1)^2$ and degree $r = 5(90k^2-20k+1)$ with $\tau = 10k(25k-4)$ and $\theta = 5(5k-1)(10k-1)$.
Case 6. ($t=6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k-6$ and $\lambda_3 = -k$, $\tau-\theta = 7k-6$, $\delta = 9k-6$, $r = 6p$ and $\theta = 6p-8k^2+6k$. Using (2.8) we find that $3p-5 = 4k(3k-4)$. Replacing k with $3k+2$ we arrive at $p = (2k+1)(18k+7)$. So we obtain that G is a strongly regular graph of order $n = 4(9k+4)^2$ and degree $r = 6(2k+1)(18k+7)$ with $\tau = 3(48k^2+45k+10)$ and $\theta = 2(3k+1)(24k+11)$.
Case 7. ($t=7$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k-7$ and $\lambda_3 = -k$, $\tau-\theta = 7k-7$, $\delta = 9k-7$, $r = 7p$ and $\theta = 7p-8k^2+7k$. Using (2.8) we find that $7p-21 = 4k(9k-14)$. Replacing k with $7k$ we arrive at $p = 252k^2-56k+3$. So we obtain that G is a strongly regular graph of order $n = 28(9k-1)^2$ and degree $r = 7(252k^2-56k+3)$ with $\tau = 14(7k-1)(14k-1)$ and $\theta = 7(7k-1)(28k-3)$.
Case 8. ($t=8$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k-8$ and $\lambda_3 = -k$, $\tau-\theta = 7k-8$, $\delta = 9k-8$, $r = 8p$ and $\theta = 8p-8k^2+8k$. Using (2.8) we find that $p = (k-1)(9k-7)$. Replacing k with $k+1$ we arrive at $p = k(9k+2)$. So we obtain that G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = 8k(9k+2)$ with $\tau = 64k^2+15k-1$ and $\theta = 8k(8k+1)$. □

Remark 2.10. We note that the complete bipartite graph $K_{5,5}$ is a strongly regular graph with $m_2 = 8m_3$. It is obtained from the class Theorem 2.4 ($\overline{9}^0$) for $k=0$.

Remark 2.11. We note that $\overline{4K_7}$ is a strongly regular graph with $m_2 = 8m_3$. It is obtained from the class Theorem 2.4 ($\overline{11}^0$) for $k=0$.

Remark 2.12. We note that $\overline{8K_8}$ is a strongly regular graph with $m_2 = 8m_3$. It is obtained from the class Theorem 2.4 ($\overline{4}^0$) for $k = 1$.

Theorem 2.4. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 8m_3$ or $m_3 = 8m_2$. Then G is one of the following strongly regular graphs:*

(1^0) *G is the complete bipartite graph $K_{5,5}$ of order $n = 10$ and degree $r = 5$ with $\tau = 0$ and $\theta = 5$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 8$ and $m_3 = 1$;*

(2^0) *G is the strongly regular graph $\overline{4K_7}$ of order $n = 28$ and degree $r = 21$ with $\tau = 14$ and $\theta = 21$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 24$ and $m_3 = 3$;*

(3^0) *G is the strongly regular graph $\overline{8K_8}$ of order $n = 64$ and degree $r = 56$ with $\tau = 48$ and $\theta = 56$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -8$ with $m_2 = 56$ and $m_3 = 7$;*

(4^0) *G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = k(9k-2)$ with $\tau = k^2+6k-1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 8k-1$ and $\lambda_3 = -k$ with $m_2 = k(9k-2)$ and $m_3 = 8k(9k-2)$;*

($\overline{4}^0$) *G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = 8k(9k-2)$ with $\tau = 64k^2-15k-1$ and $\theta = 8k(8k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k-1$ and $\lambda_3 = -8k$ with $m_2 = 8k(9k-2)$ and $m_3 = k(9k-2)$;*

(5^0) *G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = k(9k+2)$ with $\tau = k^2-6k-1$ and $\theta = k(k+1)$, where $k \geq 7$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(8k+1)$ with $m_2 = 8k(9k+2)$ and $m_3 = k(9k+2)$;*

($\overline{5}^0$) *G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = 8k(9k+2)$ with $\tau = 64k^2+15k-1$ and $\theta = 8k(8k+1)$, where $k \geq 7$. Its eigenvalues are $\lambda_2 = 8k$ and $\lambda_3 = -(k+1)$ with $m_2 = k(9k+2)$ and $m_3 = 8k(9k+2)$;*

(6^0) *G is a strongly regular graph of order $n = 4(9k-4)^2$ and degree $r = 3(2k-1)(18k-7)$ with $\tau = 18k(2k-1)$ and $\theta = (3k-2)(12k-5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 24k-11$ and $\lambda_3 = -(3k-1)$ with $m_2 = (2k-1)(18k-7)$ and $m_3 = 8(2k-1)(18k-7)$;*

($\overline{6}^0$) *G is a strongly regular graph of order $n = 4(9k-4)^2$ and degree $r = 6(2k-1)(18k-7)$ with $\tau = 3(48k^2-45k+10)$ and $\theta = 2(3k-1)(24k-11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k-2$ and $\lambda_3 = -(24k-10)$ with $m_2 = 8(2k-1)(18k-7)$ and $m_3 = (2k-1)(18k-7)$;*

(7^0) *G is a strongly regular graph of order $n = 4(9k+4)^2$ and degree $r = 3(2k+1)(18k+7)$ with $\tau = 18k(2k+1)$ and $\theta = (3k+2)(12k+5)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 3k+1$ and $\lambda_3 = -(24k+11)$ with $m_2 = 8(2k+1)(18k+7)$ and $m_3 = (2k+1)(18k+7)$;*

($\overline{7}^0$) *G is a strongly regular graph of order $n = 4(9k+4)^2$ and degree $r = 6(2k+1)(18k+7)$ with $\tau = 3(48k^2+45k+10)$ and $\theta = 2(3k+1)(24k+11)$, where*

$k \geq 0$. Its eigenvalues are $\lambda_2 = 24k+10$ and $\lambda_3 = -(3k+2)$ with $m_2 = (2k+1)(18k+7)$ and $m_3 = 8(2k+1)(18k+7)$;

(8^0) *G is a strongly regular graph of order $n = 10(9k-1)^2$ and degree $r = 4(90k^2-20k+1)$ with $\tau = 160k^2-55k+3$ and $\theta = 20k(8k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k-1$ and $\lambda_3 = -(40k-4)$ with $m_2 = 8(90k^2-20k+1)$ and $m_3 = 90k^2-20k+1$;*

($\overline{8}^0$) *G is a strongly regular graph of order $n = 10(9k-1)^2$ and degree $r = 5(90k^2-20k+1)$ with $\tau = 10k(25k-4)$ and $\theta = 5(5k-1)(10k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 40k-5$ and $\lambda_3 = -5k$ with $m_2 = 90k^2-20k+1$ and $m_3 = 8(90k^2-20k+1)$;*

(9^0) *G is a strongly regular graph of order $n = 10(9k+1)^2$ and degree $r = 4(90k^2+20k+1)$ with $\tau = 160k^2+55k+3$ and $\theta = 20k(8k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 40k+4$ and $\lambda_3 = -(5k+1)$ with $m_2 = 90k^2+20k+1$ and $m_3 = 8(90k^2+20k+1)$;*

($\overline{9}^0$) *G is a strongly regular graph of order $n = 10(9k+1)^2$ and degree $r = 5(90k^2+20k+1)$ with $\tau = 10k(25k+4)$ and $\theta = 5(5k+1)(10k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(40k+5)$ with $m_2 = 8(90k^2+20k+1)$ and $m_3 = 90k^2+20k+1$;*

(10^0) *G is a strongly regular graph of order $n = 28(9k-1)^2$ and degree $r = 2(252k^2-56k+3)$ with $\tau = 112k^2-63k+5$ and $\theta = 14k(8k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k-1$ and $\lambda_3 = -(56k-6)$ with $m_2 = 8(252k^2-56k+3)$ and $m_3 = 252k^2-56k+3$;*

($\overline{10}^0$) *G is a strongly regular graph of order $n = 28(9k-1)^2$ and degree $r = 7(252k^2-56k+3)$ with $\tau = 14(7k-1)(14k-1)$ and $\theta = 7(7k-1)(28k-3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 56k-7$ and $\lambda_3 = -7k$ with $m_2 = 252k^2-56k+3$ and $m_3 = 8(252k^2-56k+3)$;*

(11^0) *G is a strongly regular graph of order $n = 28(9k+1)^2$ and degree $r = 2(252k^2+56k+3)$ with $\tau = 112k^2+63k+5$ and $\theta = 14k(8k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 56k+6$ and $\lambda_3 = -(7k+1)$ with $m_2 = 252k^2+56k+3$ and $m_3 = 8(252k^2+56k+3)$;*

($\overline{11}^0$) *G is a strongly regular graph of order $n = 28(9k+1)^2$ and degree $r = 7(252k^2+56k+3)$ with $\tau = 14(7k+1)(14k+1)$ and $\theta = 7(7k+1)(28k+3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(56k+7)$ with $m_2 = 8(252k^2+56k+3)$ and $m_3 = 252k^2+56k+3$.*

Proof. Firstly, according[2] to Remark 2.4 we have $\alpha(\beta-1) = 8(\alpha-1)$, from which we find that $\alpha = 2$, $\beta = 5$ or $\alpha = 4$, $\beta = 7$ or $\alpha = 8$, $\beta = 8$. In view of this[3] we

[2]The all results presented in this work are verified by using a computer program `srgpar.exe`, which has been written by the author in the programming language `Borland C++ Builder 5.5`.

[3]One can use the web page `https://www.win.tue.nl/~aeb/graphs/srg/srgtab.html` that contains the parameters of strongly regular graphs from 5 upto 1300 vertices.

obtain the strongly regular graphs represented in Theorem 2.4 (1^0), (2^0), (3^0). Next, according to Proposition 2.7 it turns out that G belongs to the class $(\overline{4}^0)$ or (5^0) or $(\overline{6}^0)$ or (7^0) or (8^0) or $(\overline{9}^0)$ or (10^0) or $(\overline{11}^0)$ if $m_2 = 8m_3$. According to Proposition 2.8 it turns out that G belongs to the class (4^0) or $(\overline{5}^0)$ or (6^0) or $(\overline{7}^0)$ or $(\overline{8}^0)$ or (9^0) or $(\overline{10}^0)$ or (11^0) if $m_3 = 8m_2$. $\square$

Remark 2.13. We note that for some values of the parameter k it is possible that there exist no strongly regular graph. In view of this, we in this work describe the all feasible parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5,6,7,8$.

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